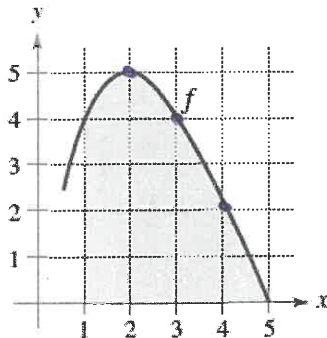


Q 36 , Sec 5.2

1. The graph of $y = f(x)$ is shown below. Using rectangles of width 1, the **upper sum** for the region bounded by the graph of f and the x -axis on the interval $[1, 5]$ is equal to

(Note: $f(4) = 2$)

- (a) 16
(b) 11
(c) 15
(d) 10
(e) 17
- upper sum $S = [f(2) + f(2) + f(3) + f(4)] \Delta x$ (correct)
 $= (5 + 5 + 4 + 2) \cdot 1$
 $= 16$

~ Q 67 , sec 5.2

2. Using the **Midpoint Rule** with $n = 3$, the approximate area of the region bounded by the graph of $f(x) = \ln x$ and the x -axis over the interval $[1, 7]$ is equal to

- (a) $2 \ln(48)$
(b) $\ln(48)$
(c) $2 \ln(105)$
(d) $2 \ln(15)$
(e) $\ln(105)$
- $\Delta x = \frac{b-a}{n} = \frac{7-1}{3} = 2$
 $M_3 = [f(2) + f(4) + f(6)] \Delta x$
 $= (\ln 2 + \ln 4 + \ln 6) \cdot 2$
 $= 2 \ln(48)$
- 1 2 3 4 5 6 7
 ✓ ✓ ✓

Q18, ch5 Problem Solving

$$3. \int_0^{\pi/4} \frac{1}{\sin^2 x + 4 \cos^2 x} dx = \int_0^{\pi/4} \frac{\sec^2 x}{\tan^2 x + 4} dx, \quad \text{let } u = \tan x \Rightarrow du = \sec^2 x dx$$

$$x=0 \Rightarrow u=0$$

$$x=\frac{\pi}{4} \Rightarrow u=1$$

$$(a) \frac{1}{2} \tan^{-1} \left(\frac{1}{2} \right)$$

$$(b) 2 \tan^{-1} (2)$$

$$(c) \tan^{-1} (2)$$

$$(d) 2 \tan^{-1} \left(\frac{1}{2} \right)$$

$$(e) \tan^{-1} \left(\frac{1}{2} \right)$$

$$= \int_0^1 \frac{du}{u^2 + 4}$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{u}{2} \right) \Big|_0^1$$

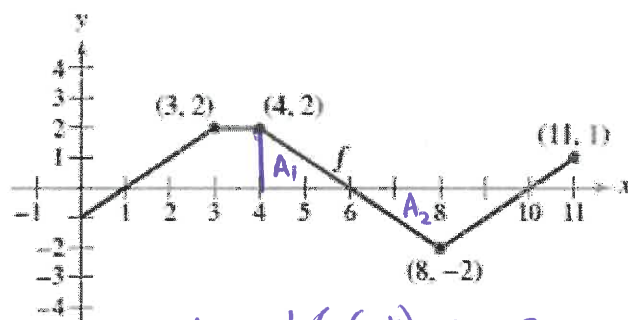
$$= \frac{1}{2} \left(\tan^{-1} \left(\frac{1}{2} \right) - \tan^{-1} 0 \right)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{1}{2} \right)$$

(correct)

Q52 part f, sec 5.3

4. If the graph of $y = f(x)$ consists of line segments, as shown below, then $\int_4^{10} f(x) dx =$



$$A_1 = \frac{1}{2} (6-4) \cdot 2 = 2$$

$$A_2 = \frac{1}{2} (10-6) \cdot 2 = 4$$

$$(a) -2$$

$$(b) -6$$

$$(c) 6$$

$$(d) -10$$

$$(e) 12$$

$$\int_4^{10} f(x) dx = \text{Net Area}$$

$$= A_1 - A_2$$

$$= 2 - 4 = -2$$

(correct)

~ Q 48, sec 5.4

5. The value of c guaranteed by the Mean Value Theorem for integrals for the function $f(x) = 2 - e^x$ over the interval $[0, 3]$ is

 f is cont. on $[0, 3]$

(a) $\ln\left(\frac{e^3 - 1}{3}\right)$

then by MVT for integrals, there is

(correct)

(b) $\frac{7 - e^3}{3}$

a number c in $[a, b]$ such that

$f(c) = f_{\text{ave}}$

(c) $\ln(e^3 - 5)$

(d) $\ln(e^3 - 4)$

(e) $3 - \ln(3)$

$$\Rightarrow 2 - e^c = \frac{1}{3} \int_0^3 (2 - e^x) dx = \frac{1}{3} [2x - e^x]_0^3 = \frac{1}{3} [6 - e^3 - (-1)]$$

$$\Rightarrow 2 - e^c = \frac{1}{3} (7 - e^3)$$

$$\Rightarrow 6 - 3e^c = 7 - e^3$$

$$\Rightarrow -3e^c = 1 - e^3$$

$$\Rightarrow e^c = \frac{e^3 - 1}{3}$$

$$\Rightarrow c = \ln\left(\frac{e^3 - 1}{3}\right) \in [0, 3]$$

~ Q 85, sec 5.4

6. If $F(x) = \int_{2\sqrt{x}}^x \sin(t^2) dt$, then $F'(4) =$

$$F'(x) = \sin(x^2) \cdot 1 - \sin(4x) \cdot 2 \cdot \frac{1}{2\sqrt{x}}$$

(a) $\frac{1}{2} \sin(16)$

$$F'(4) = \sin(16) - \sin(16) \cdot \frac{1}{2}$$

(correct)

(b) 0

(c) $\cos(16)$

(d) $\sin(16)$

(e) $\frac{1}{2} \cos(16)$

$$= \frac{1}{2} \sin(16)$$

~ Q67 and Ex5, Sec 5.5

7. $\int 3x^2 \sqrt{3x-1} dx = I$, let $u = 3x-1 \Rightarrow \frac{u+1}{3} = x$
 $\Rightarrow \frac{1}{3} du = dx$

(a) $\frac{2}{63}(3x-1)^{7/2} + \frac{4}{45}(3x-1)^{5/2} + \frac{2}{27}(3x-1)^{3/2} + C$ _____ (correct)

(b) $\frac{1}{9}(3x-1)^{5/2} + \frac{2}{9}(3x-1)^{3/2} + \frac{1}{9}(3x-1)^{1/2} + C$

(c) $\frac{2}{7}(3x-1)^{7/2} + \frac{4}{5}(3x-1)^{5/2} + \frac{2}{3}(3x-1)^{3/2} + C$

(d) $\frac{2}{63}(3x-1)^{7/2} + \frac{4}{5}(3x-1)^{5/2} + \frac{2}{3}(3x-1)^{3/2} + C$

(e) $\frac{1}{9}(3x-1)^{7/2} + \frac{2}{9}(3x-1)^{5/2} + \frac{1}{9}(3x-1)^{3/2} + C$

$$I = \int \cancel{3} \left(\frac{u+1}{3}\right)^2 \sqrt{u} \cdot \frac{1}{3} du = \frac{1}{9} \int (u^2 + 2u + 1) u^{1/2} du$$

$$= \frac{1}{9} \int (u^{5/2} + 2u^{3/2} + u^{1/2}) du$$

$$= \frac{1}{9} \left[\frac{2}{7} u^{7/2} + 2 \cdot \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right] + C$$

$$= \frac{2}{63} (3x-1)^{7/2} + \frac{4}{45} (3x-1)^{5/2} + \frac{2}{27} (3x-1)^{3/2} + C$$

Q 88, Sec 5.5

8. $\int_{\pi/12}^{\pi/4} \csc(2x) \cot(2x) dx = I$, let $u = 2x \Rightarrow du = 2 dx$

$x = \frac{\pi}{12} \Rightarrow u = \frac{\pi}{6}$

$x = \frac{\pi}{4} \Rightarrow u = \frac{\pi}{2}$

(a) $\frac{1}{2}$ _____ (correct)

(b) $\frac{1}{2} - \frac{1}{\sqrt{3}}$

(c) $-\frac{1}{2}$

(d) $-\frac{1}{2} + \frac{1}{\sqrt{3}}$

(e) -1

$$I = \int_{\pi/6}^{\pi/2} \csc u \cot u \frac{du}{2}$$

$$= -\frac{1}{2} \csc u \Big|_{\pi/6}^{\pi/2}$$

$$= -\frac{1}{2} (1 - 2) = \frac{1}{2}$$

~ Q33, Sec 5.7

9. $\int_0^{\pi/2} \tan\left(\frac{x}{3}\right) dx = I,$ let $u = \frac{x}{3} \Rightarrow du = \frac{1}{3} dx$
 $x=0 \Rightarrow u=0$
 $x=\frac{\pi}{2} \Rightarrow u=\frac{\pi}{6}$

(a) $3 \ln\left(\frac{2}{\sqrt{3}}\right)$

(b) $3 \ln(2)$

(c) $\frac{1}{3} \ln\left(\frac{2}{\sqrt{3}}\right)$

(d) $\frac{1}{3} \ln(2)$

(e) $\ln\left(\frac{\sqrt{3}}{2}\right)$

$I = \int_0^{\pi/6} 3 \tan u \, du$
 $= 3 \ln|\sec u| \Big|_0^{\pi/6}$
 $= 3 \left(\ln\left(\frac{2}{\sqrt{3}}\right) - \ln 1 \right)$
 $= 3 \ln\left(\frac{2}{\sqrt{3}}\right)$ (correct)

~ Ex 5, Sec 5.7

10. $\int \frac{3x^2 + x + 27}{x^2 + 9} dx = \int \left(3 + \frac{x}{x^2 + 9} \right) dx$
 $= 3x + \frac{1}{2} \ln(x^2 + 9) + C$

$$\begin{array}{r} 3 \\ x^2 + 9 \overline{) 3x^2 + x + 27} \\ \underline{3x^2 } \\ x + 27 \end{array}$$

(a) $3x + \ln \sqrt{x^2 + 9} + C$ (correct)

(b) $3x + \ln(x^2 + 9) + C$

(c) $3x + \frac{1}{3} \tan^{-1}\left(\frac{x}{3}\right) + C$

(d) $3x + \tan^{-1}\left(\frac{x}{3}\right) + C$

(e) $(x^3 + \frac{1}{2}x^2 + 27x) \ln(x^2 + 9) + C$

~ EX 5, sec 5.8

11. $\int_{5/2}^{15/4} \frac{1}{\sqrt{5x - x^2}} dx = I,$ Note: $5x - x^2 = -(x^2 - 5x)$
 $= -(x^2 - 5x + \frac{25}{4}) + \frac{25}{4}$
 $= \frac{25}{4} - (x - \frac{5}{2})^2$

(a) $\frac{\pi}{6}$ (correct)
 (b) $\frac{\pi}{3}$
 (c) $\frac{2\pi}{3}$
 (d) $\frac{\pi}{4}$
 (e) $\frac{\pi}{2}$

$I = \int_{5/2}^{15/4} \frac{dx}{\sqrt{\frac{25}{4} - (x - \frac{5}{2})^2}}$ let $u = x - \frac{5}{2} \Rightarrow du = dx$
 $x = \frac{5}{2} \Rightarrow u = 0$
 $x = \frac{15}{4} \Rightarrow u = \frac{5}{4}$

$= \int_0^{5/4} \frac{du}{\sqrt{(\frac{5}{2})^2 - u^2}}$
 $= \left[\sin^{-1}\left(\frac{u}{5/2}\right) \right]_0^{5/4}$
 $= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1} 0$
 $= \frac{\pi}{6}$

~ Q 49 sec 5.9

12. $\int_2^{2-\ln 4} \sinh^2\left(1 - \frac{x}{2}\right) \cosh\left(1 - \frac{x}{2}\right) dx = I,$
 let $u = \sinh\left(1 - \frac{x}{2}\right) \Rightarrow du = \cosh\left(1 - \frac{x}{2}\right) \cdot \left(-\frac{1}{2}\right) dx$

$x = 2 \Rightarrow u = \sinh 0 = 0$
 $x = 2 - \ln 4 \Rightarrow u = \sinh\left(1 - \frac{2 - \ln 4}{2}\right)$ (correct)
 $= \sinh(\ln 2) = \frac{e^{\ln 2} - e^{-\ln 2}}{2}$
 $= \frac{2 - 1/2}{2} = \frac{3}{4}$

$I = \int_0^{3/4} -2u^2 du$
 $= -2 \left[\frac{u^3}{3} \right]_0^{3/4} = -\frac{2}{3} \left(\frac{3}{4} \right)^3 = -\frac{9}{32}$

(a) $-\frac{9}{32}$
 (b) $-\frac{27}{32}$
 (c) $-\frac{125}{96}$
 (d) $-\frac{9}{64}$
 (e) $-\frac{3}{8}$

Q35, Sec 7.1

13. The area of the region bounded by the graphs of

$$f(x) = \frac{1}{1+x^2} \text{ and } g(x) = \frac{1}{2}x^2$$

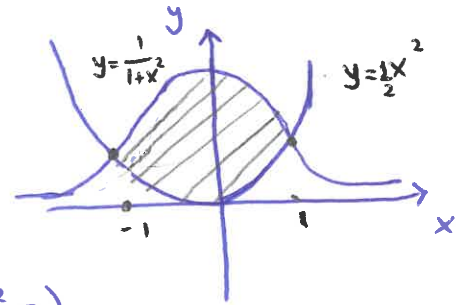
is equal to

intersection Points: $\frac{1}{1+x^2} = \frac{x^2}{2}$

$$\Rightarrow 2 = x^2 + x^4$$

$$\Rightarrow x^4 + x^2 - 2 = 0 \Rightarrow (x^2 - 1)(x^2 + 2) = 0$$

$$x = \pm 1$$



(a) $\frac{\pi}{2} - \frac{1}{3}$

(b) $\frac{\pi}{2} - \frac{1}{6}$

(c) $\frac{\pi}{4} - \frac{1}{6}$

(d) $\frac{\pi}{4} - \frac{1}{3}$

(e) $\frac{\pi}{4} - \frac{2}{3}$

$$\begin{aligned} A &= \int_{-1}^1 \left(\frac{1}{1+x^2} - \frac{1}{2}x^2 \right) dx \\ &= 2 \int_0^1 \left(\frac{1}{1+x^2} - \frac{1}{2}x^2 \right) dx \\ &= 2 \left[\tan^{-1} x - \frac{1}{2} \frac{x^3}{3} \right]_0^1 \\ &= 2 \left[\left(\frac{\pi}{4} - \frac{1}{6} \right) - 0 \right] \\ &= \frac{\pi}{2} - \frac{1}{3} \end{aligned}$$

(correct)

~ Ex3, sec 7.1

14. The area of the region bounded by the graphs of

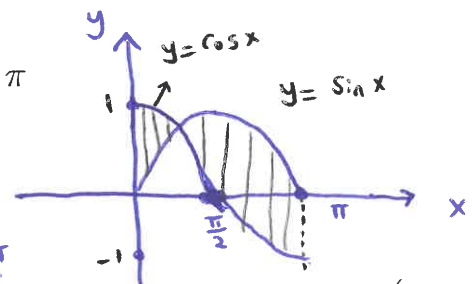
$$f(x) = \sin x, \quad g(x) = \cos x, \quad x = 0, \text{ and } x = \pi$$

is equal to

intersection point:

$$\sin x = \cos x \Rightarrow \tan x = 1$$

$$\Rightarrow x = \frac{\pi}{4} \in [0, \pi]$$



(a) $2\sqrt{2}$

(b) $2\sqrt{2} - 2$

(c) 2

(d) $2\sqrt{2} - 1$

(e) $2\sqrt{2} + 2$

$$\begin{aligned} A &= \int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} (\sin x - \cos x) dx \\ &= \left[\sin x + \cos x \right]_0^{\pi/4} + \left[-\cos x - \sin x \right]_{\pi/4}^{\pi} \\ &= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \right) - (0+1) + (1-0) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \\ &= 2\sqrt{2} \end{aligned}$$

(correct)

