Q	MASTER	1	2	3	4	5	6	7	8
1	A	B 2	B 1	C 8	E 2	B 4	D 10	C 6	D 7
2	A	C 3	B 14	D 10	B 4	E 3	E 7	B 14	E 12
3	A	C 9	A 4	E 11	E 1	D 6	A 13	A 2	E 4
4	A	B 7	E 7	C 14	E 5	D 7	B 9	C 10	B 10
5	A	C 11	D 12	A 4	E 8	E 5	E 11	A 11	B 8
6	A	E 6	E 10	B 7	A 3	D 8	E 6	B 12	D 11
7	A	B 13	E 3	C 6	D 6	C 9	B 4	A 3	D 1
8	A	B 8	D 13	C 9	C 10	B 12	A 14	D 13	C 3
9	A	D 10	A 11	B 2	A 9	D 10	C 3	B 8	A 5
10	A	A 1	B 5	D 12	E 14	A 1	B 1	B 5	A 9
11	A	A 5	B 8	C 3	C 13	D 2	B 8	B 9	B 14
12	A	C 12	A 9	E 13	A 12	E 13	E 12	B 1	D 6
13	A	A 4	D 6	D 5	A 7	E 11	B 5	E 7	A 2
14	A	A 14	A 2	A 1	E 11	D 14	A 2	B 4	B 13

Key Solution



~ Q4 / Sec 7.3

1. The volume of the solid generated by revolving the region bounded by the graphs of $y = 1 - x$, $y = 0$, and $x = 3$ about the line $y = 1$ is equal to

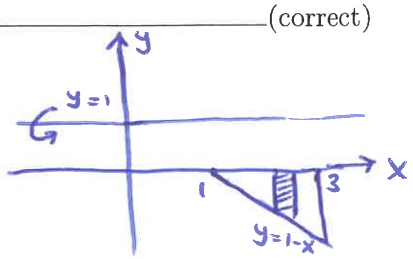
(a) $\frac{20\pi}{3}$ (correct)

(b) $\frac{4\pi}{3}$

(c) $\frac{40\pi}{3}$

(d) 6π

(e) 2π



Washer with thickness $= \Delta x$

$R = 1 - 0 = 1$

$r = 1 - (1-x) = x$

$$V = \pi \int_a^b (R^2 - r^2)\, dx$$

$$= \pi \int_1^3 (x^2 - 1^2)\, dx = \pi \left[\frac{x^3}{3} - x \right]_1^3$$

$$= \pi \left[(9-3) - \left(\frac{1}{3} - 1 \right) \right] = \pi \left(7 - \frac{1}{3} \right) = \frac{20\pi}{3}$$

~ EX3 / Sec 7.3

2. The volume of the solid generated by revolving the region bounded by the graphs of $y = 2x^2 + 2$, $y = 0$, $x = 0$, and $x = 2$ about the y-axis is equal to
(Hint: The shell method is preferable.)

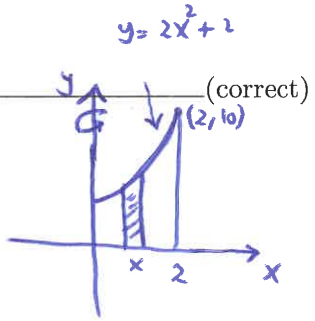
(a) 24π (correct)

(b) 12π

(c) 20π

(d) 16π

(e) 8π



Shell with thickness $= dx$

$R = x - 0 = x$

$H = 2x^2 + 2 - 0 = 2x^2 + 2$

$$V = 2\pi \int_a^b R\, H\, dx$$

$$= 2\pi \int_0^2 x(2x^2 + 2)\, dx = 2\pi \int_0^2 (2x^3 + 2x)\, dx$$

$$= 2\pi \left[\frac{x^4}{2} + x^2 \right]_0^2 = 2\pi (8 + 4) = 24\pi$$

Q20, Sec 7.4

3. The arc length of the graph of $y = \frac{1}{3}x^{3/2} - x^{1/2}$ from $x = 1$ to $x = 4$ is equal to

$y' = \frac{1}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$ is cont for $1 \le x \le 4$

(a) $\frac{10}{3}$ — (correct)

(b) $\frac{4}{3}$

(c) $\frac{14}{3}$

(d) $\frac{2}{3}$

(e) $\frac{16}{3}$

$(y')^2 = \frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1}$

$1 + (y')^2 = \frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1} = \left(\frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2}\right)^2$

Now, $L = \int_1^4 \sqrt{1+(y')^2}\,dx = \int_1^4 \sqrt{\left(\frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2}\right)^2}\,dx$

$= \int_1^4 \left(\frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2}\right)dx$

$= \frac{1}{2}\cdot\frac{2}{3}x^{3/2} + \frac{1}{2}\cdot 2\,x^{1/2}\Big]_1^4$

$= \left(\frac{8}{3} + 2\right) - \left(\frac{1}{3} + 1\right)$

$= \frac{7}{3} + 1 = \frac{10}{3}$

~ Ex6, Sec 7.4

4. The area of the surface generated by revolving the curve $y = x^3$ on the interval $[0, 2]$ about the x-axis is equal to

$y' = 3x^2$ is cont for $0 \le x \le 2$

(a) $\frac{\pi}{27}\left(145\sqrt{145} - 1\right)$ — (correct)

(b) $\frac{\pi}{54}\left(145\sqrt{145} - 1\right)$

(c) $\frac{\pi}{9}\left(145\sqrt{145} - 1\right)$

(d) $\frac{\pi}{36}\left(145\sqrt{145} - 1\right)$

(e) $\frac{\pi}{18}\left(145\sqrt{145} - 1\right)$

$S = 2\pi \int_0^2 y\sqrt{1+(y')^2}\,dx$

$= 2\pi \int_0^2 x^3\sqrt{1+9x^4}\,dx$

let $u = 1 + 9x^4 \Rightarrow du = 36x^3\,dx$

$x = 0 \Rightarrow u = 1$, $x = 2 \Rightarrow u = 145$

$= \frac{2\pi}{36}\int_1^{145} u^{1/2}\,du$

$= \frac{\pi}{18}\cdot\frac{2}{3}u^{3/2}\Big]_1^{145}$

$= \frac{\pi}{27}\left(145\sqrt{145} - 1\right)$

~ Q34, Sec 8.1

5. $\int \frac{4}{3-e^{-x}}\,dx =$ $\int \frac{4e^x}{3e^x-1}\,dx$, let $u = 3e^x-1 \Rightarrow du = 3e^x\,dx$

$= \frac{4}{3}\int \frac{du}{u} = \frac{4}{3}\ln|u| + C$

(a) $\frac{4}{3}\ln|3e^x-1|+C$ (correct)

(b) $4\ln|3-e^{-x}|+C$

(c) $\frac{3}{4}\ln|3e^x-1|+C$

(d) $4\ln|3e^x-1|+C$

(e) $-4\ln|3e^x-1|+C$

$= \frac{4}{3}\ln|3e^x-1| + C$

~ EX 7, Sec 8.2

6. $\int x^2\cos(4x)\,dx =$ $\frac{1}{4}x^2\sin(4x) + \frac{1}{8}x\cos(4x) - \frac{1}{32}\sin(4x) + C$

(a) $\frac{1}{4}x^2\sin(4x)+\frac{1}{8}x\cos(4x)-\frac{1}{32}\sin(4x)+C$ (correct)

(b) $\frac{1}{4}x^2\sin(4x)+\frac{1}{8}x\cos(4x)+\frac{1}{32}\sin(4x)+C$

(c) $\frac{1}{4}x^2\sin(4x)-\frac{1}{8}x\cos(4x)-\frac{1}{32}\sin(4x)+C$

(d) $\frac{1}{4}x^2\sin(4x)+\frac{1}{16}x\cos(4x)+\frac{1}{64}\sin(4x)+C$

(e) $\frac{1}{4}x^2\sin(4x)-\frac{1}{16}x\cos(4x)-\frac{1}{64}\sin(4x)+C$

u	dv
x^2	$\cos(4x)$
$2x$	$\frac{1}{4}\sin(4x)$
2	$-\frac{1}{16}\cos(4x)$
0	$-\frac{1}{64}\sin(4x)$

(⊕, ⊖, ⊕)

~ Q17 , Problem Solving (ch8)

7. Suppose that $f(0) = 1$, $f(1) = 2$, and $\int_0^1 f(x)\,dx = 4$. If the second derivative of f is continuous on the closed interval $[0, 1]$, then $\int_0^1 (x^2 - x)f''(x)\,dx =$ I

(a) 5 (correct)

(b) 1

(c) 9

(d) 11

(e) 7

$u = x^2 - x \qquad dv = f''(x)\,dx$

$du = (2x-1)\,dx \qquad v = f'(x)$

$I = (x^2-x) f'(x) \Big]_0^1 + \int_0^1 (1-2x)\, f'(x)\,dx$

$u = 1-2x \qquad dv = f'(x)\,dx$

$du = -2\,dx \qquad v = f(x)$

$= 0 + (1-2x) f(x) \Big]_0^1 + 2\int_0^1 f(x)\,dx$

$= -f(1) - f(0) + 2\int_0^1 f(x)\,dx$

$= -2 - 1 + 2(4) = 5$

~ EX4 , Sec 8.3

8. $\int \frac{\tan^3 x}{\sec x}\,dx =$

$\int \frac{\tan^2 x \cdot \tan x \sec x\,dx}{\sec^2 x} = \int \frac{(\sec^2 x - 1)\cdot \sec x \tan x\,dx}{\sec^2 x}$

let $u = \sec x \Rightarrow du = \sec x \tan x\,dx$

(a) $\sec x + \cos x + C$ (correct)

(b) $\frac{1}{2}\sec^2 x + \cos x + C$

(c) $\sec x + \sin x + C$

(d) $\sec x - \cos x + C$

(e) $\csc x - \sin x + C$

$= \int \frac{u^2 - 1}{u^2}\,du$

$= \int (1 - u^{-2})\,du$

$= u - \frac{u^{-1}}{-1} + C$

$= \sec x + \cos x + C$

Q47, Sec 8.3

9. $\int_0^{\pi/4} \sin(x)\sin(3x)\,dx =$ $\frac{1}{2}\int_0^{\pi/4}\left[\underbrace{\cos(-2x)}_{=\cos(2x)} - \cos(4x)\right]dx$

(a) $\frac{1}{4}$ (correct)

(b) $\frac{1}{8}$

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

(e) $-\frac{3}{2}$

$= \frac{1}{2}\left[\frac{1}{2}\sin(2x) - \frac{1}{4}\sin(4x)\right]_0^{\pi/4}$

$= \frac{1}{2}\left[\left(\frac{1}{2}-0\right) - 0\right]$

$= \frac{1}{4}$

~ Q23, Sec 8.4

10. $\int \frac{dx}{x\sqrt{x^2+9}} = I$,

let $x = 3\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 3\sec^2\theta\,d\theta$

$\sqrt{x^2+9} = \sqrt{9\tan^2\theta+9} = 3\sec\theta$

(a) $-\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}+3}{x}\right| + C$ (correct)

(b) $-\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}-3}{x}\right| + C$

(c) $\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}+x}{3}\right| + C$

(d) $\ln\left|\frac{\sqrt{x^2+9}+3}{x}\right| + C$

(e) $-\ln\left|\frac{\sqrt{x^2+9}+3}{x}\right| + C$

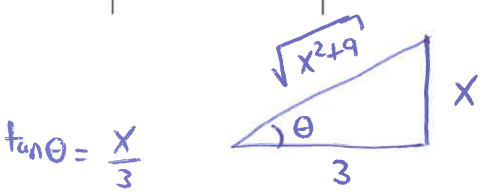
So, $I = \int \frac{3\sec^2\theta\,d\theta}{3\tan\theta\cdot 3\sec\theta}$

$= \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\,d\theta = \frac{1}{3}\int \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\,d\theta$

$= \frac{1}{3}\int \csc\theta\,d\theta$

$= \frac{-1}{3}\ln|\csc\theta + \cot\theta| + C$

$= -\frac{1}{3}\ln\left|\frac{\sqrt{x^2+9}}{x} + \frac{3}{x}\right| + C$



$\tan\theta = \frac{x}{3}$

Q27, Sec 8.4 $u = e^x$

11. $\int e^x \sqrt{1-e^{2x}}\,dx =$ $\int \sqrt{1-u^2}\,du$, let $u = \sin\theta$, $0 < \theta < \frac{\pi}{2}$ why? $u = e^x > 0$

$du = \cos\theta\,d\theta$

$\sqrt{1-u^2} = \cos\theta$

$= \int \cos^2\theta\,d\theta = I.$

(a) $\frac{1}{2}\left(\sin^{-1}(e^x) + e^x\sqrt{1-e^{2x}}\right) + C$ (correct)

(b) $-\frac{1}{2}\left(\sin^{-1}(e^x) + e^x\sqrt{1-e^{2x}}\right) + C$

(c) $\frac{1}{2}\left(\sin^{-1}(e^x) - e^x\sqrt{1-e^{2x}}\right) + C$

(d) $\frac{1}{2}\left(\sin^{-1}(e^x) - 2e^x\sqrt{1-e^{2x}}\right) + C$

(e) $\frac{1}{2}\left(2\sin^{-1}(e^x) + e^x\sqrt{1-e^{2x}}\right) + C$

So, $I = \int \cos^2\theta\,d\theta$

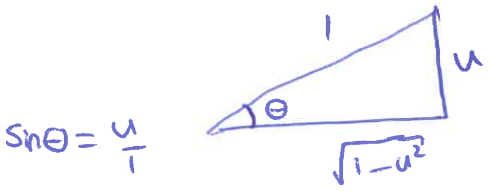
$= \frac{1}{2}\int [1+\cos(2\theta)]\,d\theta$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin(2\theta)\right] + C$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\cdot 2\sin\theta\cos\theta\right] + C$

$= \frac{1}{2}\left[\sin^{-1}u + \frac{u}{1}\cdot\frac{\sqrt{1-u^2}}{1}\right] + C$

$= \frac{1}{2}\left[\sin^{-1}(e^x) + e^x\sqrt{1-e^{2x}}\right] + C$



$\sin\theta = \frac{u}{1}$

~ Q23, Sec 8.5

12. $\int \frac{x+16}{x(x^2+4)}\,dx = I.$

Consider $\frac{x+16}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$\Rightarrow x+16 = A\cdot(x^2+4) + Bx^2 + Cx$

(a) $4\ln|x| - 2\ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ (correct)

(b) $4\ln|x| - 2\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C$

(c) $4\ln|x| + 2\ln(x^2+4) + \frac{1}{2}\tan^{-1}(x) + C$

(d) $4\ln|x| - 4\ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C$

(e) $4\ln|x| - 4\ln(x^2+4) + \tan^{-1}(x) + C$

$x = 0$: $16 = 4A \Rightarrow A = 4$

Coeff of x^2: $0 = A + B \Rightarrow B = -4$

Coeff of x: $1 = C$

split

So, $I = \int \frac{4}{x}\,dx + \int \frac{-4x}{x^2+4}\,dx + \int \frac{1}{x^2+4}\,dx$

let $u = x^2+4$

$= 4\ln|x| - 2\ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C_1$

~ Q13 , Sec 8.5

13. If $\dfrac{4x+4}{x(x-2)^2} = \dfrac{A}{x} + \dfrac{B}{x-2} + \dfrac{C}{(x-2)^2}$, then $A+B+C=$

$\Rightarrow \quad 4x+4 = A\cdot(x-2)^2 + B\cdot x\cdot(x-2) + C\,x$

(a) 6 (correct)

(b) 8

(c) 4

(d) 5

(e) 7

$x=0 \;:\; 4 = 4A \Rightarrow A=1$

$x=2 \;:\; 12 = 2C \Rightarrow C=6$

Coeff of x^2: $0 = A+B \Rightarrow B=-1$

So, $A+B+C = 6$

~ Q57, Sec 8.7 $\quad t=\tan\left(\frac{x}{2}\right),\ -\pi<x<\pi$

14. $\displaystyle\int \frac{dx}{1-\sin x+\cos x} \overset{\uparrow}{=} \int \frac{\frac{2\,dt}{1+t^2}}{1-\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}} = \int \frac{2\,dt}{1+t^2-2t+1-t^2}$

$= \displaystyle\int \frac{2\,dt}{2-2t}$

$= \displaystyle\int \frac{1}{1-t}\,dt$

$= -\ln|1-t| + C$

$= -\ln\left|1-\tan\left(\frac{x}{2}\right)\right| + C$

(a) $-\ln\left|1-\tan\left(\frac{x}{2}\right)\right|+C$ (correct)

(b) $\ln\left|1-\tan\left(\frac{x}{2}\right)\right|+C$

(c) $-\ln\left|1+\tan\left(\frac{x}{2}\right)\right|+C$

(d) $-\frac{1}{2}\ln\left|1-\tan\left(\frac{x}{2}\right)\right|+C$

(e) $-\ln\left|2-\tan\left(\frac{x}{2}\right)\right|+C$