

MATH 106 Major Exam I (Term 261)
Master Version

1. Let

$$f(z) = \frac{z^2 - az - 4}{z^2 + b}.$$

If $\lim_{z \rightarrow 0} f(z) = -4$ and $f(1) = -4$, then $a + b =$

Section 10.1 – Exact Exercise 18

(a) 6 **(Correct Answer)**

(b) 1

(c) 5

(d) -4

(e) -5

Solution

From

$$\lim_{z \rightarrow 0} f(z) = \frac{-4}{b} = -4,$$

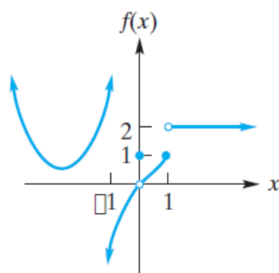
we get $b = 1$. Also,

$$f(1) = \frac{1 - a - 4}{1 + b} = \frac{-a - 3}{2} = -4,$$

so $-a - 3 = -8$, hence $a = 5$. Therefore,

$$a + b = 5 + 1 = \boxed{6}.$$

2. Given the graph of f shown below,



which of the following statements is true?

Section 10.2 – Exercise 1

(a) $\lim_{x \rightarrow -\infty} f(x) = +\infty$ (**Correct Answer**)

(b) $\lim_{x \rightarrow -\infty} f(x) = -\infty$

(c) $\lim_{x \rightarrow 0^-} f(x) = 1$

(d) $\lim_{x \rightarrow 1^-} f(x) = 2$

(e) $\lim_{x \rightarrow 0^+} f(x) = 1$

Solution

From the left branch of the graph, $f(x)$ increases without bound as $x \rightarrow -\infty$. Thus

$$\lim_{x \rightarrow -\infty} f(x) = +\infty.$$

Also, the graph approaches 0 from both sides as $x \rightarrow 0$, while as $x \rightarrow 1^-$ it approaches 1. Hence the other statements are false.

3. Let

$$h(x) = \frac{x-3}{x^3-9x}.$$

The number of discontinuities of h is

Section 10.3 – Exercise 25

(a) 3 **(Correct Answer)**

(b) 1

(c) 2

(d) 0

(e) 4

Solution

Factor the denominator:

$$x^3 - 9x = x(x-3)(x+3).$$

Thus the denominator is zero at $x = -3, 0, 3$. Even though $x-3$ cancels algebraically, the original function is still undefined at $x = 3$. Hence there are

$$\boxed{3}$$

discontinuities.

4. Let

$$f(x) = \begin{cases} ax+3, & x < 2, \\ x^2-a, & x \geq 2. \end{cases}$$

If f is continuous at $x = 2$, then $a =$

Section 10.3 – Similar to Exercises 29–34

(a) $\frac{1}{3}$ **(Correct Answer)**

(b) $-\frac{1}{3}$

(c) 1

(d) $\frac{3}{2}$

(e) 3

Solution

Continuity at $x = 2$ requires the two expressions to agree there:

$$2a + 3 = 2^2 - a = 4 - a.$$

Therefore

$$3a = 1 \quad \implies \quad \boxed{a = \frac{1}{3}}.$$

5. The number of points on the curve

$$y = \frac{5}{2}x^2 - x^3$$

at which the tangent line is horizontal is

Section 11.2 – Exercise 85

(a) 2 (**Correct Answer**)

(b) 1

(c) 3

(d) 0

(e) 4

Solution

A horizontal tangent occurs when $y' = 0$. Since

$$y' = 5x - 3x^2 = x(5 - 3x),$$

we have

$$x = 0 \quad \text{or} \quad x = \frac{5}{3}.$$

Therefore there are $\boxed{2}$ such points.

6. The total revenue from selling q units of a product is

$$r(q) = 180q + 6q^2 - \frac{1}{3}q^3.$$

The marginal revenue at $q = 5$ is greater than the marginal revenue at $q = 10$ by

Section 11.3 – Similar to Exercises 23–26

(a) 15 (**Correct Answer**)

(b) 10

(c) 20

(d) 25

(e) 5

Solution

The marginal-revenue function is

$$r'(q) = 180 + 12q - q^2.$$

Hence

$$r'(5) = 180 + 60 - 25 = 215, \quad r'(10) = 180 + 120 - 100 = 200.$$

Thus the difference is

$$215 - 200 = \boxed{15}.$$

7. The equation of the tangent line to the graph of

$$f(x) = \frac{4}{2x - 1}$$

at $x = 1$ is

Section 11.1 – Similar to Exercises 27–28

(a) $y = -8x + 12$ (**Correct Answer**)

(b) $y = 8x - 4$

(c) $y = -8x + 4$

(d) $y = 8x + 12$

(e) $y = -4x + 8$

Solution

First,

$$f(1) = \frac{4}{2(1) - 1} = 4.$$

Also,

$$f'(x) = -\frac{8}{(2x - 1)^2}, \quad f'(1) = -8.$$

Using the point-slope form,

$$y - 4 = -8(x - 1),$$

so

$$\boxed{y = -8x + 12}.$$

8. If the consumption function is

$$C = 6 + \frac{3I}{4} - \frac{\sqrt{I}}{3},$$

then marginal propensity to consume when $I = 25$ is

Section 11.4 – Exact Exercise 66

(a) $\frac{43}{60}$ (**Correct Answer**)

(b) $\frac{17}{60}$

(c) $\frac{3}{4}$

(d) $\frac{7}{10}$

(e) $\frac{1}{30}$

Solution

The marginal propensity to consume is dC/dI :

$$\frac{dC}{dI} = \frac{3}{4} - \frac{1}{6\sqrt{I}}.$$

At $I = 25$,

$$\frac{dC}{dI} = \frac{3}{4} - \frac{1}{30} = \frac{45}{60} - \frac{2}{60} = \boxed{\frac{43}{60}}.$$

9. If

$$f(x) = \frac{x^2 + 1}{x + 1},$$

then $f'(1) =$

Section 11.4 – Similar to Exercises 44 and 46

- (a) $\frac{1}{2}$ **(Correct Answer)**
- (b) 1
- (c) $\frac{3}{2}$
- (d) $\frac{1}{4}$
- (e) 2

Solution

By the quotient rule,

$$f'(x) = \frac{2x(x+1) - (x^2+1)}{(x+1)^2}.$$

Therefore

$$f'(1) = \frac{2(1)(2) - 2}{4} = \boxed{\frac{1}{2}}.$$

10. The cost c of producing q units of a product is given by

$$c = 4000 + 10q + 0.1q^2.$$

If the price per unit p is related to q by

$$q = 600 - 2p,$$

then the rate of change of cost with respect to price per unit when $p = 100$ is

(Hint: use the chain rule.)

Section 11.5 – Similar to Exercise 71

- (a) -180 **(Correct Answer)**
- (b) 200
- (c) -90
- (d) 90
- (e) -200

Solution

Using the chain rule,

$$\frac{dc}{dp} = \frac{dc}{dq} \frac{dq}{dp}.$$

Here

$$\frac{dc}{dq} = 10 + 0.2q, \quad \frac{dq}{dp} = -2.$$

When $p = 100$,

$$q = 600 - 2(100) = 400.$$

Therefore

$$\frac{dc}{dp} = (10 + 0.2(400))(-2) = 90(-2) = \boxed{-180}.$$

11. If

$$y = (2t - 1)^3,$$

then $\frac{dy}{dt}$ at $t = 1$ is

Section 11.5 – Similar to Exercise 51

(a) 6 (**Correct Answer**)

(b) 3

(c) 12

(d) 2

(e) 9

Solution

By the chain rule,

$$\frac{dy}{dt} = 3(2t - 1)^2(2) = 6(2t - 1)^2.$$

At $t = 1$,

$$\frac{dy}{dt} = 6(1)^2 = \boxed{6}.$$

12. If

$$y = \ln(x^2 + 1)^3,$$

then $y' =$

Section 12.1 – Similar to Exercise 23

(a) $\frac{6x}{x^2 + 1}$ (**Correct Answer**)

(b) $\frac{3x}{x^2 + 1}$

(c) $\frac{6x}{(x^2 + 1)^2}$

(d) $6x \ln(x^2 + 1)$

(e) $\frac{3}{x^2 + 1}$

Solution

Using the logarithm power property,

$$y = 3 \ln(x^2 + 1).$$

Hence

$$y' = 3 \left(\frac{2x}{x^2 + 1} \right) = \boxed{\frac{6x}{x^2 + 1}}.$$

13. If the total-cost function is

$$c = 25 \ln(q + 1) + 12,$$

then the marginal cost when $q = 6$ is

Section 12.1 – Exercise 49

(a) $\frac{25}{7}$ **(Correct Answer)**

(b) $\frac{25}{6}$

(c) $\frac{7}{25}$

(d) $\frac{25}{49}$

(e) 25

Solution

The marginal cost is

$$\frac{dc}{dq} = \frac{25}{q + 1}.$$

At $q = 6$,

$$\frac{dc}{dq} = \boxed{\frac{25}{7}}.$$

14. If

$$f(x) = 5^{x^2 \ln x},$$

then $f'(1) =$

Section 12.2 – Exercise 30

(a) $\ln 5$ **(Correct Answer)**

(b) $5 \ln 5$

(c) 1

(d) 5

(e) $2 \ln 5$

Solution

Let

$$u = x^2 \ln x.$$

Then

$$u' = 2x \ln x + x.$$

Since $f(x) = 5^u$,

$$f'(x) = 5^u \ln 5 \, u'.$$

At $x = 1$, $u = 0$ and $u' = 1$. Hence

$$f'(1) = 5^0 \ln 5 = \boxed{\ln 5}.$$

15. If

$$y^3 = 4x,$$

then the slope of the curve at the point $(2, 2)$ is

Section 12.4 – Exercise 8

- (a) $\frac{1}{3}$ **(Correct Answer)**
- (b) $\frac{2}{3}$
- (c) $\frac{3}{4}$
- (d) $\frac{4}{3}$
- (e) 3

Solution

Differentiate implicitly:

$$3y^2 \frac{dy}{dx} = 4.$$

Therefore

$$\frac{dy}{dx} = \frac{4}{3y^2}.$$

At $(2, 2)$,

$$\frac{dy}{dx} = \frac{4}{3(2)^2} = \boxed{\frac{1}{3}}.$$