1. Suppose $y = Ce^{mx}$ is a solution of the differential equation

$$y'' + y' - 6y = 0.$$

If $y(0) = 2$ and $\lim_{x\to\infty} y(x) = 0$, then y is

(a) $y = 2e^{-3x}$ _____ (correct)
(b) $y = -2e^{-2x}$
(c) $y = 2e^{-2x}$
(d) $y = 2e^{3x}$
(e) $y = -2e^{-3x}$

2. A particle moves along a straight line with acceleration

$$a(t) = \frac{1}{t^2}, \qquad t > 0.$$

If $s(t)$ denotes the position of the particle, $s(1) = 0$, and $s'(1) = 1$, then $s(t)$ is

(a) $s(t) = 2t - \ln t - 2$ _____ (correct)
(b) $s(t) = \ln t$
(c) $s(t) = 2t + \frac{1}{t} - 3$
(d) $s(t) = t + (\ln t)^2 - 1$
(e) $s(t) = t - (\ln t)^2 - 1$

3. The solution of the initial-value problem

$$\frac{dy}{dx} = \frac{1+\sin x}{y-1}, \qquad y(0) = 2,$$

is

(a) $y = 1 + \sqrt{2x - 2\cos x + 3}$ (correct)

(b) $y = 1 + \sqrt{2x + 2\cos x - 1}$

(c) $y = 1 + \sqrt{2x - \cos x + 2}$

(d) $y = 1 + \sqrt{2x - 4\cos x + 5}$

(e) $y = 1 + \sqrt{2x - 2\cos x + 1}$

4. A hot cup of coffee is placed in a room whose temperature is 25°C. The temperature of the coffee is initially 85°C, and after 10 minutes it is 55°C. Using Newton's law of cooling, the temperature $T(t)$ after t minutes is

(a) $T(t) = 25 + 60\left(\frac{1}{2}\right)^{t/10}$ (correct)

(b) $T(t) = 25 + 60\left(\frac{1}{3}\right)^{t/10}$

(c) $T(t) = 25 + 60\left(\frac{1}{2}\right)^{10t}$

(d) $T(t) = 25 + 60\left(\frac{2}{3}\right)^{t/10}$

(e) $T(t) = 25 + 60\left(\frac{1}{2}\right)^{t/5}$

5. The solution of the initial-value problem

$$(x^2+1)y' + 2xy = xe^x, \qquad y(0) = 1$$

is

(a) $y = \dfrac{(x-1)e^x + 2}{x^2+1}$ (correct)

(b) $y = \dfrac{xe^x + 1}{x^2+1}$

(c) $y = \dfrac{(x+1)e^x}{x^2+1}$

(d) $y = \dfrac{(x-1)e^x + 1}{x^2+1}$

(e) $y = \dfrac{(x-2)e^x + 2}{x^2+1}$

6. A one-parameter family of solutions of the differential equation

$$y' = -4x^2 + 4xy - y^2 + 2$$

is given by

(a) $y = 2x + \dfrac{1}{x+C}$ (correct)

(b) $y = 2x - \dfrac{1}{x+C}$

(c) $y = 2x + \dfrac{1}{C-x}$

(d) $y = x + \dfrac{1}{x+C}$

(e) $y = 2x + \dfrac{1}{2(x+C)}$

7. A general nonzero solution of the differential equation

$$x^2y' = xy + 3y^2$$

is

(a) $\dfrac{x}{y} + 3\ln|x| = C$ (correct)

(b) $\dfrac{y}{x} + 3\ln|x| = C$

(c) $\dfrac{x}{y} - 3\ln|x| = C$

(d) $\dfrac{3x}{y} + \ln|x| = C$

(e) $\dfrac{x}{y} + 3\ln|y| = C$

8. By an appropriate substitution, the Bernoulli equation

$$x\sqrt[3]{y^2}y' + 5\sqrt[3]{y^5} = x^3$$

can be transformed into the linear equation

(a) $3xv' + 25v = 5x^3$ (correct)

(b) $3xv' - 25v = -5x^3$

(c) $3xv' + 5v = x^3$

(d) $3xv' - 25v = 5x^3$

(e) $3xv' - 5v = x^3$

9. A general nonzero solution of the differential equation

$$2xy' + y^3 e^{-2x} = 2xy$$

is given by

(a) $y^{-2} = e^{-2x}\big(\ln|x| + C\big)$ ——— (correct)
(b) $y^{-2} = e^{2x}\big(\ln|x| + C\big)$
(c) $y^{-2} = e^{-2x}\big(-\ln|x| + C\big)$
(d) $y^{2} = e^{-2x}\big(\ln|x| + C\big)$
(e) $y^{-2} = e^{-2x}\left(\frac{1}{2}\ln|x| + C\right)$

10. If $k(0) = 2$ and the differential equation

$$\big(2y\cos 2x + 5x^4y^2\big)\,dx + \big(k(x) + 2x^5y\big)\,dy = 0$$

is exact, then $k(x)$ is

(a) $2 + \sin 2x$ ——— (correct)
(b) $2 - \sin 2x$
(c) $\sin 2x$
(d) $2 + 2\sin 2x$
(e) $2 + \cos 2x$

11. The general solution of the differential equation

$$\left(x^3 + \frac{y}{x}\right)\,dx + \left(y^2 + \ln x\right)\,dy = 0$$

is

(a) $3x^4 + 12y\ln x + 4y^3 = C$ (correct)

(b) $x^4 + y\ln x + \dfrac{y^3}{3} = C$

(c) $\dfrac{x^4}{4} + y\ln x + y^3 = C$

(d) $\dfrac{x^4}{4} + x\ln|y| + \dfrac{y^3}{3} = C$

(e) $\dfrac{x^4}{4} + y\ln x + \dfrac{y^2}{2} = C$

12. If a nonconstant solution of the differential equation

$$yy'' = (y')^2 + y'$$

has tangent line $y = 2x + 1$ at $x = 0$, then the solution is given by

(a) $y = \dfrac{1}{3} + \dfrac{2}{3}e^{3x}$ (correct)

(b) $y = \dfrac{1}{2} + \dfrac{1}{2}e^{4x}$

(c) $y = e^{2x}$

(d) $y = -1 + 2e^{x}$

(e) $y = 3 - 2e^{-x}$

13. If

$$a\begin{bmatrix}1\\2\end{bmatrix}+b\begin{bmatrix}1\\-1\end{bmatrix}+c\begin{bmatrix}-1\\0\end{bmatrix}+d\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix},$$

and every solution (a,b,c,d) can be written in the form

$$s(1,0,p,q)+t(0,1,r,l),$$

then the value of $\dfrac{p+r}{q+l}$ is

(a) -2 (correct)
(b) 2
(c) -1
(d) 1
(e) 0

14. Which one of the following subsets is a subspace of $\mathbb{R}^4$?

(a) The set of all vectors (x_1,x_2,x_3,x_4) such that $x_1+x_2+x_3=0$ and $2x_2-x_4=0$ (correct)
(b) The set of all vectors (x_1,x_2,x_3,x_4) such that $x_1^2+x_2=0$
(c) The set of all vectors (x_1,x_2,x_3,x_4) such that $x_1-3x_2+x_4=2$
(d) The set of all vectors (x_1,x_2,x_3,x_4) such that $x_1\leq x_2$
(e) The set of all vectors (x_1,x_2,x_3,x_4) such that $x_1x_4=0$

15. The vectors $u = (1, 0, k)$, $v = (0, 1, 1)$, and $w = (4 - k, 1, 4)$ in $\mathbb{R}^3$ are linearly independent when

(a) $k \neq 1, 3$ (correct)

(b) $k = 1$ or $k = 3$

(c) $k \neq -1, -3$

(d) $k = -1$ or $k = -3$

(e) $k \neq -1, 3$