

1. If each of the following systems is inconsistent

$$3x + y + az = 1$$

$$2x + y - z = 2$$

$$-x + 2y + 3z = 0$$

$$4x + y - z = 3$$

$$-2x - y + bz = -3$$

$$2x - y + z = 1$$

.

then $a + b =$

(a) -1 _____(correct)

(b) 0

(c) 1

(d) 2

(e) -2

2. If A is an $n \times n$ idempotent matrix (*i.e.* $A^2 = A$), and I is the $n \times n$ identity matrix, then $(I - A)^{101} =$

(a) $I - A$ _____(correct)

(b) A

(c) $99A$

(d) $101A$

(e) $100A$

3. If the LU factorization of $A = \begin{bmatrix} -2 & 1 & 2 \\ 4 & 1 & -2 \\ -6 & -3 & 4 \end{bmatrix}$ is given by

$$A = LU = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & a & 1 \end{bmatrix} \begin{bmatrix} c & d & 2 \\ 0 & e & 2 \\ 0 & 0 & 2 \end{bmatrix},$$

then $a + b + c + d + e =$

- (a) 3 _____(correct)
- (b) 2
- (c) 1
- (d) -1
- (e) -2

4. If A and B are 3×3 matrices with $\det(A^T) = -1$ and $\det(B) = 2$, then $\det(2AB^{-1}) =$

- (a) -4 _____(correct)
- (b) -2
- (c) $-\frac{1}{4}$
- (d) $-\frac{1}{2}$
- (e) 0

5. Consider the following subset of $\mathbb{R}^3$:

$$A := \{(x, y, z) \in \mathbb{R}^3 \mid 2x - y + z = 0\},$$

$$B := \{(x, y, z) \in \mathbb{R}^3 \mid x^2 = y + z\},$$

$$C := \{(x, y, z) \in \mathbb{R}^3 \mid y + z = 0\}.$$

- (a) A and C are subspace of $\mathbb{R}^3$ while B is not _____(correct)
- (b) B is a subspace of $\mathbb{R}^3$ while A, C are not
- (c) C is a subspace of $\mathbb{R}^3$ while A, B are not
- (d) A and B are subspace of $\mathbb{R}^3$ while C is not
- (e) A is a subspace of $\mathbb{R}^3$ while B, C are not

6. The null space of $\begin{bmatrix} 1 & 3 & -4 \\ 2 & -1 & -1 \\ -1 & -3 & 4 \end{bmatrix}$ is spanned by

(a) $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ _____(correct)

(b) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

7. Consider the set $S := \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} \subseteq \mathbb{R}^4$, where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

- (a) S is a basis for $\mathbb{R}^4$ _____(correct)
- (b) S is linearly independent but not a spanning set for $\mathbb{R}^4$
- (c) S is a spanning set for $\mathbb{R}^4$ but not linearly independent
- (d) S is neither linearly independent nor a spanning set for $\mathbb{R}^4$
- (e) S spans a subspace of dimension 3 of $\mathbb{R}^4$

8. A homogeneous linear system of 3 equations in 4 variables

- (a) has always an infinite number of solutions _____(correct)
- (b) may be inconsistent
- (c) may have a unique solution
- (d) may have exactly four solutions
- (e) has at most two solutions

9. Given a vector space V , consider $\{\mathbf{0}\}$
(the set consisting of one element only which is the zero vector in V).

- (a) $\{\mathbf{0}\}$ is a subspace of V and has dimension 0. _____(correct)
- (b) $\{\mathbf{0}\}$ is a subspace of V and has dimension 1.
- (c) $\{\mathbf{0}\}$ is a subspace of V and has no basis.
- (d) $\{\mathbf{0}\}$ is a subspace of V and has no dimension.
- (e) $\{\mathbf{0}\}$ is not a subspace of V .

10. Let $V = C^2[0, 1]$, $S = \{e^x, e^{-x}, \sinh(x)\}$ and $W = \text{Span}(S)$.

- (a) $\dim(W) = 2$ _____(correct)
- (b) $\dim(W) = 3$
- (c) $\dim(W) = 1$
- (d) S is linearly independent
- (e) S is a spanning set of V .

11. Consider the sets $\mathbb{R}$ of real numbers and $\mathbb{Q}$ of rational numbers.

- (a) $\mathbb{R}$ is an infinite dimensional vector space over $\mathbb{Q}$. _____(correct)
- (b) $\mathbb{R}$ is a finite dimensional vector space over $\mathbb{Q}$.
- (c) $\mathbb{Q}$ is an infinite dimensional vector space over $\mathbb{R}$.
- (d) $\mathbb{Q}$ is a finite dimensional vector space over $\mathbb{R}$.
- (e) Neither $\mathbb{R}$ is a vector space over $\mathbb{Q}$ nor $\mathbb{Q}$ is a vector space over $\mathbb{R}$.

12. Let A, B be $n \times n$ matrices such that $B = E_1 E_3 E'_1 A$, where E_1, E'_1 are $n \times n$ elementary matrices of the first type, and E_3 is an $n \times n$ elementary matrix of the third type.

- (a) $\det(B) = \det(A)$ _____(correct)
- (b) $\det(B) = -\det(A)$
- (c) $\det(B) = 2 \det(A)$
- (d) $\det(B) = -2 \det(A)$
- (e) $\det(B) = 3 \det(A)$