King Fahd University of Petroleum & Minerals

Department of Mathematics **MATH 513-03 – Major Exam 1 – Term 261**

Date: 27 September 2026 **Time Allowed: 120 Minutes** **Instructor: Dr. Adel Al-Mahdi**

Name: _____ ID No.: _____

- Mobile phones, calculators, and other electronic devices are not allowed.
- Show all necessary steps clearly; unsupported answers may receive no credit.
- Simplify your answers whenever possible and clearly box your final results.

Question 1. [10 marks]

Use the eigenvalue–eigenvector method to find the real-valued general solution of the system

$$\begin{cases} x_1' = -2x_1 - 13x_2, \\ x_2' = x_1 + 4x_2. \end{cases}$$

Question 2. [15 marks]

Let f be the 2π-periodic function defined by

$$f(t) = \begin{cases} -t, & -\pi \le t \le 0, \\ t, & 0 < t \le \pi. \end{cases}$$

(a) Find the Fourier series of f.

(b) By evaluating the Fourier series at a suitable point, show that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}.$$

(c) Use Parseval's equality to show that

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}.$$

Question 3. [10 marks]

Find the Fourier sine series of

$$f(t) = e^{2t}, \qquad 0 < t < \pi.$$

Question 4. [10 marks]

Find the complex Fourier series of the 2π-periodic function defined on $(-\pi, \pi)$ by

$$f(t) = \begin{cases} 1, & 0 < t < \pi, \\ -1, & -\pi < t < 0. \end{cases}$$

Question 5. [15 marks]

Let f be the 2π-periodic function defined by

$$f(t) = \begin{cases} 0, & -\pi \le t \le 0, \\ t, & 0 < t \le \dfrac{\pi}{2}, \\ \pi - t, & \dfrac{\pi}{2} < t \le \pi. \end{cases}$$

Its Fourier series is

$$f(t) = \frac{\pi}{8} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{\cos\left(2(2n-1)t\right)}{(2n-1)^2} - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)^2} \sin\left((2n-1)t\right).$$

(a) By differentiating the given Fourier series term by term, obtain a Fourier series representation for $f'(t)$.

(b) Apply the Fourier convergence theorem to the series obtained in part (a) at $t = 0$ to evaluate the infinite sum

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}.$$