King Fahd University of Petroleum and Minerals

College of Computing and Mathematics

Department of Mathematics

STAT 511 – Midterm Exam

Term 261 **Date: Oct 10, 2026**

Time Allowed: 120 Minutes **Total Marks: 68**

MASTER VERSION – the correct answer to every multiple-choice item is option A

Name: _____

ID number: _____ **Section:** _____

Instructions

- Answer all three questions. Question 1 contains 14 multiple-choice items; Questions 2 and 3 are written problems.
- Textbooks, notes, and mobile phones are not allowed. A nonprogrammable scientific calculator is permitted.
- For Question 1, select **one** answer (A–E) per item and record its letter in the grid below. Each correct answer earns 3 marks. Incorrect, blank, unclear or multiple answers earn zero marks.
- **A formula sheet** with critical values is attached at the end of the paper.

Multiple-choice answer grid

Item	1	2	3	4	5	6	7
Answer							

Item	8	9	10	11	12	13	14
Answer							

Question	**Marks Awarded**	**Maximum Marks**
1. Multiple choice, items 1–14		42
2. Randomized complete block design		13
3. Factorial design		13
Total		**68**

Question 1. Multiple choice

42 marks; 3 marks per item

Record one letter for each item in the answer grid on page 1.

1. Sixteen randomly selected bottles of a carbonated beverage have mean shelf life $\bar{y} = 131$ days and standard deviation $S = 20$ days. Assuming a normal population, the 95% confidence interval for the mean shelf life is

A. [120.35, 141.66] days
B. [121.20, 140.80] days
C. [88.38, 173.62] days
D. [122.23, 139.77] days
E. [126.00, 136.00] days

Work space

2. Six steel specimens are each measured once by Method 1 and once by Method 2.

Specimen	1	2	3	4	5	6
Method 1	12	15	11	14	16	13
Method 2	10	14	8	14	14	9

Assume independent, normally distributed differences $d = \text{Method 1} - \text{Method 2}$. Which test statistic and degrees of freedom are appropriate for $H_0 : \mu_d = 0$?

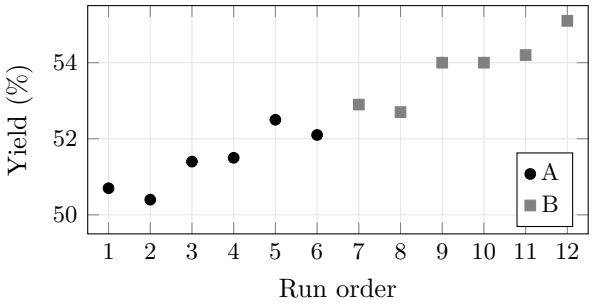
A. $t_0 = 3.46$ with 5 df
B. $t_0 = 1.41$ with 5 df
C. $t_0 = 3.46$ with 10 df
D. $t_0 = 1.45$ with about 8.7 df (Welch)
E. $t_0 = 8.49$ with 5 df

Work space

3. In the Portland cement mortar study, the 95% pooled-t confidence interval for $\Delta = \mu_{\text{modified}} - \mu_{\text{unmodified}}$ (tension bond strength) is $[-0.545, -0.011]$ kgf/cm^2. Which statement is correct?

A. The two-sided pooled t-test of $H_0 : \Delta = 0$ at $\alpha = 0.05$ rejects H_0; the data indicate a lower mean bond strength for the modified mortar.
B. There is a 95% probability that the fixed value Δ lies in $[-0.545, -0.011]$.
C. About 95% of individual modified specimens are between 0.011 and 0.545 kgf/cm^2 weaker than unmodified specimens.
D. Because the interval excludes zero, the reduction has been shown to be important in engineering practice.
E. The two-sided p-value must exceed 0.05, because the interval is narrow.

4. Two treatments with the *same* true mean yield are compared with six runs each. All A runs were performed first, followed by all B runs. The plot shows the observed yield against run order.



The analyst reports that "B is better by about 2.4 points." What is the best explanation and remedy?

A. An upward drift over time is aligned with the treatments because run order was not randomized; randomize and record run order in a new experiment.
B. The experiment was not blocked; adding more runs of B would remove the bias.
C. Measurement error is too large; each run should be measured several times and the readings averaged.
D. Treatment B has a larger variance, so Welch's t-test should be used.
E. The errors are not normal; a normal Q–Q plot of the residuals would reveal the problem.

5. Three oven temperatures are compared using four separate rack/oven runs per temperature, in random order. Each rack is inspected five times; an analyst incorrectly treats all 60 readings as independent observations. When the one-way ANOVA instead uses one mean reading per rack, what are the experimental unit and error degrees of freedom?

A. One rack (oven run); 9 df
B. One inspection reading; 57 df
C. One rack (oven run); 57 df
D. One temperature setting; 2 df
E. One inspection reading; 48 df

6. For a balanced two-treatment comparison, the run-to-run standard deviation is $\sigma \approx 4$ units and the smallest important difference is $\delta = 2$ units. Using the planning approximation $n \approx 16/(\delta/\sigma)^2$ runs per treatment (two-sided 5% test, 80% power), how many runs per treatment are needed? If the budget allows only 16 runs per treatment, what is the smallest difference δ that can be detected with the same power?

A. $n = 64$; $\delta = 4$
B. $n = 4$; $\delta = 1$
C. $n = 8$; $\delta = 2$
D. $n = 16$; $\delta = 4$
E. $n = 64$; $\delta = 8$

Work space

7. A one-way ANOVA of four catalysts ($n = 5$ runs each) is significant, with $MS_E = 10$ on 16 df. The treatment means are:

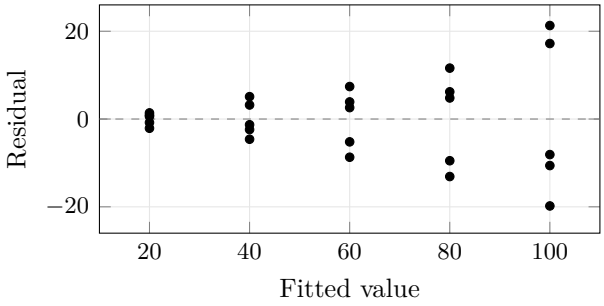
Catalyst	1	2	3	4
Mean response	20.0	23.0	27.5	28.0

Using Fisher's LSD with $\alpha = 0.05$, which pairs of catalysts differ significantly?

A. Only 1–3, 1–4, 2–3, and 2–4.
B. All six pairs.
C. Only 1–4.
D. Only 1–3 and 1–4.
E. Only 1–2, 1–3, 1–4, and 2–4.

Work space

8. The plot shows the residuals against the fitted values from a one-way ANOVA with five treatments. What does it indicate?



A. The error variance increases with the mean response (nonconstant variance).
B. A single outlier; the other residuals are satisfactory.
C. A time trend: the errors are not independent.
D. A structureless horizontal band: all assumptions are satisfied.
E. The errors are not normal, because the points do not lie on a straight line.

9. An RCBD compares $a = 4$ extrusion pressures in $b = 6$ resin batches. The fitted RCBD gives $SS_{\text{Block}} = 150$ and $SS_E = 75$. If the analyst *ignored* the batches and fitted a treatment-only (one-way) model to the same 24 observations, what would the error mean square be?

A. 11.25
B. 5.00
C. 15.00
D. 7.50
E. 35.00

Work space

10. Four formulations (Latin letters) are tested on four days (rows) by four operators (columns), and each run uses one of four machines (Greek letters), in the layout below.

	Op. 1	Op. 2	Op. 3	Op. 4
Day 1	$A\alpha$	$B\beta$	$C\gamma$	$D\delta$
Day 2	$D\beta$	$C\alpha$	$B\delta$	$A\gamma$
Day 3	$B\gamma$	$A\delta$	$D\alpha$	$C\beta$
Day 4	$C\delta$	$D\gamma$	$A\beta$	$B\alpha$

The data are analyzed with the additive Graeco-Latin square model. How many error degrees of freedom are available?

A. 3
B. 6
C. 9
D. 12
E. 0

Work space

11. Four catalysts are compared, but each raw-material batch supports only two runs. The design uses six batches (an × marks a run):

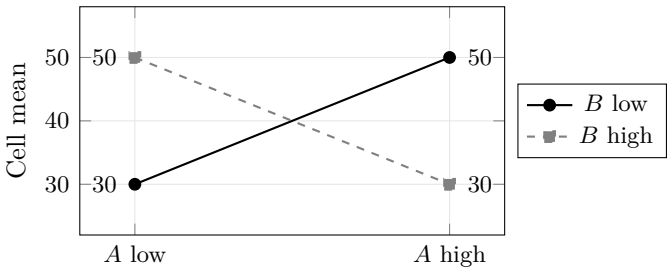
	Batch 1	Batch 2	Batch 3	Batch 4	Batch 5	Batch 6
Catalyst 1	×	×	×			
Catalyst 2	×			×	×	
Catalyst 3		×		×		×
Catalyst 4			×		×	×

For this balanced incomplete block design, what are the number of replicates per treatment r, the pair concurrence λ, and the error degrees of freedom of the intrablock ANOVA?

A. $r = 3,\ \lambda = 1,$ error df $= 3$
B. $r = 2,\ \lambda = 1,$ error df $= 3$
C. $r = 3,\ \lambda = 2,$ error df $= 5$
D. $r = 3,\ \lambda = 1,$ error df $= 6$
E. $r = 6,\ \lambda = 3,$ error df $= 3$

Work space

12. The interaction plot shows the four cell means of a replicated 2×2 factorial experiment.



Which statement is correct?

A. The estimated main effect of A is zero, but its estimated simple effects are $+20$ at low B and -20 at high B; interpret A together with B.
B. The main effect of A is zero, so A does not affect the response and can be dropped from further study.
C. Because the lines cross, the main effect of B is large ($+20$).
D. The interaction effect AB is zero, because the average of the two simple effects of A is zero.
E. The plot alone proves that the interaction is statistically significant.

13. The battery experiment has 3 materials, 3 temperatures (15, 70, 125 °F), and 4 batteries per combination ($N = 36$). The intended analysis is a full categorical 3×3 factorial ANOVA, with no linear-trend restriction. A student fits the model below and obtains this output.

```
> fit <- aov(life ~ material * temp, data = battery); summary(fit)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
material	2	10684	5342	6.259	0.00535
temp	1	39043	39043	45.743	1.69e-07
material:temp	2	2315	1158	1.356	0.27298
Residuals	30	25605	854		

What is the most likely explanation for these degrees of freedom?

A. `temp` was left numeric, so R fitted a linear temperature trend (1 df). To fit the intended categorical model, convert `temp` with `factor()`.

B. One temperature level has missing observations, so R removed one df from `temp`.

C. The interaction is not significant, so R automatically removed two df from the interaction.

D. The residual df of 30 shows that the experiment had five replicates per cell.

E. `material` was stored as a numeric variable instead of a factor.

14. A completely randomized experiment compares four coating formulations with five specimens each. The one-way ANOVA gives $SS_{\text{Treat}} = 60$ and $SS_E = 80$. The exact p-value of the F-test is not available. By comparing F_0 with upper-tail F critical values, what can be concluded about the p-value, and what are the decisions at $\alpha = 0.05$ and $\alpha = 0.01$?

A. $0.025 < p < 0.05$; reject H_0 at $\alpha = 0.05$ but not at $\alpha = 0.01$.

B. $0.05 < p < 0.10$; do not reject H_0 at $\alpha = 0.05$ or at $\alpha = 0.01$.

C. $0.01 < p < 0.025$; reject H_0 at $\alpha = 0.05$ but not at $\alpha = 0.01$.

D. $p < 0.01$; reject H_0 at both $\alpha = 0.05$ and $\alpha = 0.01$.

E. $p > 0.10$; do not reject H_0 at either level.

Work space

Question 2. Randomized complete block design **13 marks**

Four cleaning methods are compared over six days. Each day supplies four separate specimens, with one specimen randomly assigned to each method. The response is cleaning efficiency (%). Assume fixed method effects, additive day effects, and independent normal errors with a common variance.

The method means are $\bar{y}_A = 20$, $\bar{y}_B = 22$, $\bar{y}_C = 24$, and $\bar{y}_D = 26$. The corrected total SS is 360, the method SS is 120, and the day SS is 150.

Each box in the ANOVA table is worth $\frac{1}{2}$ mark. Every other box, each circled decision, and each true/false statement is worth 1 mark.

(a) Complete the ANOVA table. **(4 marks)**

Source	SS	df	MS	F_0
Methods	120			
Days	150			—
Error				—
Total	360	23	—	—

(b) Test $H_0 : \mu_A = \mu_B = \mu_C = \mu_D$ at the 5% level. **(2 marks)**

Critical value $F_{0.05;\,\nu_1,\,\nu_2} =$ ☐ Decision (circle one): **Reject** H_0 **Do not reject** H_0

(c) Calculate the **individual** 95% confidence interval for $\mu_D - \mu_A$. **(3 marks)**

Standard error = ☐ 95% CI: [☐ , ☐]

(d) Circle **T** (true) or **F** (false) for each statement. **(4 marks)**

	Statement	Answer	
(i)	If the day effects were ignored and a one-way ANOVA were fitted to the same 24 observations, the error df would be 20.	T	F
(ii)	The interval in part (c) indicates that method D has a higher mean efficiency than method A.	T	F
(iii)	If the day effect had $p = 0.42$, the day term should be deleted from this prespecified analysis.	T	F
(iv)	Rejecting H_0 in part (b) shows that all four method means differ from one another.	T	F

Question 3. Factorial design **13 marks**

A completely randomized experiment studies two resin types (A_1, A_2) and three curing temperatures (B_1, B_2, B_3), with **three independent specimens per combination**. The response is strength (MPa), and larger values are preferred. Both factors are fixed. Assume independent normal errors with a common variance. The cell means and sums of squares are given below.

Cell mean (MPa)	B_1	B_2	B_3
A_1	12	16	20
A_2	20	18	16

Each box in the ANOVA table and in part (c) is worth $\frac{1}{2}$ mark. The critical-value box, the circled decision, and each true/false statement are worth 1 mark.

(a) Complete the full factorial ANOVA table. **(5 marks)**

Source	SS	df	MS	F_0
Resin (A)	18			
Temperature (B)	12			
$A \times B$	108			
Error	72	12		—
Total	210	17	—	—

(b) Test for a resin × temperature interaction at the 5% level. **(2 marks)**

Critical value $F_{0.05;\,\nu_1,\,\nu_2} =$ ☐ Decision (circle one): **Reject** H_0 **Do not reject** H_0

(c) Calculate the estimated simple effect $A_2 - A_1$ at each temperature and the estimated marginal effect of changing from A_1 to A_2 (in MPa). **(2 marks)**

	at B_1	at B_2	at B_3	Marginal effect
$A_2 - A_1$				

(d) Circle **T** (true) or **F** (false) for each statement. **(4 marks)**

	Statement	Answer	
(i)	The marginal effect of resin alone adequately summarizes how resin affects strength across the three temperatures.	T	F
(ii)	At temperature B_3, resin A_1 has the higher observed mean strength.	T	F
(iii)	The estimated error standard deviation is $\hat{\sigma} = \sqrt{6} \approx 2.45$ MPa.	T	F
(iv)	Because A_1B_3 has one of the largest observed means (20 MPa), it is proven to be significantly stronger than every other combination.	T	F

End of exam.

Formula sheet

STAT 511 – Midterm, Term 261

1. Means, intervals, and t-tests

Sample mean	$\bar{y} = \sum_{i=1}^{n} y_i / n$
Sample variance	$S^2 = \left[\sum y_i^2 - (\sum y_i)^2/n\right]/(n-1)$
For paired data	Apply the above to $y_i = d_i$; $S_d = \sqrt{S_d^2}$.
Standard error	$\text{se}(\bar{y}) = S/\sqrt{n}$
CI for μ	$\bar{y} \pm t_{\alpha/2,\, n-1}\, S/\sqrt{n}$
One-sample test	$t_0 = (\bar{y} - \mu_0)/(S/\sqrt{n})$, df $n-1$
Paired test	$t_0 = \bar{d}/(S_d/\sqrt{n})$, df $n-1$
Pooled variance	$S_p^2 = [(n_1-1)S_1^2 + (n_2-1)S_2^2] / (n_1+n_2-2)$
Pooled test	$t_0 = (\bar{y}_1 - \bar{y}_2)/\left(S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)$, df n_1+n_2-2
Welch test	$t_0 = (\bar{y}_1 - \bar{y}_2)/\sqrt{S_1^2/n_1 + S_2^2/n_2}$

2. One-way ANOVA (a treatments, n each)

Partition	$SS_T = SS_{\text{Treat}} + SS_E$
df ($N = an$)	Treatments $a-1$; Error $N-a$; Total $N-1$
Test	$F_0 = MS_{\text{Treat}}/MS_E$, $MS = SS/\text{df}$
Error SD	$\hat{\sigma} = \sqrt{MS_E}$
R^2	SS_{Treat}/SS_T
Fisher LSD	$t_{\alpha/2,\, N-a}\sqrt{2MS_E/n}$

3. RCBD (a treatments, b blocks)

Partition	$SS_T = SS_{\text{Trt}} + SS_{\text{Block}} + SS_E$
df	Treatments $a-1$; Blocks $b-1$
	Error $\nu = (a-1)(b-1)$
CI for $\mu_i - \mu_\ell$	$(\bar{y}_{i\cdot} - \bar{y}_{\ell\cdot}) \pm t_{\alpha/2,\,\nu}\sqrt{2MS_E/b}$

4. Latin squares and incomplete blocks

Latin square ($p \times p$)	Error df $(p-1)(p-2)$
Graeco-Latin square	Error df $(p-1)(p-3)$
BIBD	$ar = bk = N$, $\lambda(a-1) = r(k-1)$
BIBD error df	$N - a - b + 1$

a: treatments; b: blocks; k: runs per block;
r: replicates per treatment; N: total runs;
λ: blocks containing each pair of treatments.

5. Factorial designs (n replicates per cell)

Two factors	A: $a-1$; B: $b-1$; AB: $(a-1)(b-1)$
	Error: $ab(n-1)$; Total: $abn-1$
Partition	$SS_T = SS_A + SS_B + SS_{AB} + SS_E$
Fixed-effect tests	$MS_j = SS_j/\text{df}_j$, $F_j = MS_j/MS_E$
	$j = A, B, AB$ (and RCBD treatments)
Three factors	ABC: $(a-1)(b-1)(c-1)$
	Error: $abc(n-1)$
2×2 main effect	$A = \bar{y}_{A+} - \bar{y}_{A-}$
2×2 interaction	$AB = \frac{1}{2}[(A \text{ at } B^+) - (A \text{ at } B^-)]$

Effects: two levels of A, b levels of B

Estimated simple effect at B_j: $\widehat{\Delta}_A(B_j) = \bar{y}_{2j\cdot} - \bar{y}_{1j\cdot}$.
Estimated marginal effect (balanced design):
$\widehat{\Delta}_A = \bar{y}_{2\cdot\cdot} - \bar{y}_{1\cdot\cdot} = b^{-1}\sum_{j=1}^{b}\widehat{\Delta}_A(B_j)$.

6. Critical values (upper-tail)

ν	5	6	8	9	10	12	15	16	18	20	24	27
$t_{0.025,\nu}$	2.571	2.447	2.306	2.262	2.228	2.179	2.131	2.120	2.101	2.086	2.064	2.052
$t_{0.05,\nu}$	2.015	1.943	1.860	1.833	1.812	1.782	1.753	1.746	1.734	1.725	1.711	1.703

(ν_1, ν_2)	$F_{0.10}$	$F_{0.05}$	$F_{0.025}$	$F_{0.01}$
(1, 12)	3.177	4.747	6.554	9.330
(2, 12)	2.807	3.885	5.096	6.927
(3, 12)	2.606	3.490	4.474	5.953
(3, 15)	2.490	3.287	4.153	5.417
(3, 16)	2.462	3.239	4.077	5.292
(4, 15)	2.361	3.056	3.804	4.893
(2, 27)	2.511	3.354	4.242	5.488
(4, 27)	2.165	2.728	3.307	4.106