King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 208
Major Exam II
Term 253
19 July 2026

EXAM COVER

Number of versions: 4
Number of questions: 15

King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 208
Major Exam II
Term 253
July 19, 2026
Net Time Allowed: 90 Minutes

MASTER VERSION

1. If the solution space of the system

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0, \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0, \\ x_1 + 3x_2 + 3x_3 + 13x_4 &= 0 \end{aligned}$$

consists of all linear combinations of the vectors $v_1 = (a, b, 1, 0)$ and $v_2 = (c, d, 0, 1)$, then $a+b+c+d =$

(a) -6 ______ (correct)
(b) 6
(c) -4
(d) 4
(e) 0

2. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) \in \mathbb{R}^4 \mid a+2b+3c+4d = 0,\ b+2c+3d = 0\}$. A basis of S consists of the vectors

(a) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$ ______ (correct)
(b) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -4, 2, 0)$
(c) $v_1 = (1, 2, 1, 0),\ v_2 = (2, -3, 0, 1)$
(d) $v_1 = (-1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$
(e) $v_1 = (2, -3, 0, 1),\ v_2 = (4, -6, 0, 2)$

3. The rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$ is

   (a) 2 ______________________________ (correct)
   (b) 3
   (c) 1
   (d) 4
   (e) 0

4. If $W(x)$ is the Wronskian of the functions

$$f_1(x) = \sin(5x) + 2\cos(5x), \text{ and } f_2(x) = 3\sin(5x) + \cos(5x),$$

   then $W(x) =$

   (a) 25 ______________________________ (correct)
   (b) -25
   (c) 5
   (d) 0
   (e) $25\cos(5x)$

5. If $y(x)$ is the solution of the initial-value problem

$$35y'' - y' - 12y = 0, \qquad y(0) = 5, \quad y'(0) = 3,$$

then $y(1) =$

(a) $5e^{3/5}$ ______________________________ (correct)
(b) $5e^{-4/7}$
(c) $3e^{3/5} + 2e^{-4/7}$
(d) $2e^{3/5} + 3e^{-4/7}$
(e) 0

6. The general solution of the differential equation $y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$ is

(a) $y(x) = \left(C_1 + C_2x + C_3x^2\right)e^{x} + C_4e^{-2x}$ ______________________________ (correct)
(b) $y(x) = \left(C_1 + C_2x\right)e^{x} + \left(C_3 + C_4x\right)e^{-2x}$
(c) $y(x) = \left(C_1 + C_2x + C_3x^2\right)e^{x} + C_4xe^{-2x}$
(d) $y(x) = C_1e^{x} + \left(C_2 + C_3x + C_4x^2\right)e^{-2x}$
(e) $y(x) = \left(C_1 + C_2x\right)e^{x} + C_3e^{-x} + C_4e^{-2x}$

7. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx)\cos 2x + (C + Dx)\sin 2x + Ee^{2x} + Fe^{-2x}$$

is

(a) $y^{(6)} + 4y^{(4)} - 16y'' - 64y = 0$ ________ (correct)
(b) $y^{(6)} - 4y^{(4)} - 16y'' + 64y = 0$
(c) $y^{(6)} + 8y^{(4)} + 16y'' = 0$
(d) $y^{(6)} - 8y^{(4)} + 16y'' = 0$
(e) $y^{(4)} - 16y = 0$

8. Given that $y = e^{4x}\cosh x$ is a solution of the differential equation

$$y^{(4)} - 12y^{(3)} + 60y'' - 164y' + 195y = 0,$$

the general solution of the differential equation is

(a) $y = C_1e^{5x} + C_2e^{3x} + e^{2x}\left(C_3\cos 3x + C_4\sin 3x\right)$ ________ (correct)
(b) $y = C_1e^{-3x} + C_2e^{-5x} + e^{2x}\left(C_3\cos 4x + C_4\sin 4x\right)$
(c) $y = C_1e^{5x} + C_2e^{3x} + e^{-2x}\left(C_3\cos 3x + C_4\sin 3x\right)$
(d) $y = C_1e^{-3x} + C_2e^{-5x} + e^{3x}\left(C_3\cos 2x + C_4\sin 2x\right)$
(e) $y = C_1e^{5x} + C_2e^{3x} + e^{x}\left(C_3\cos 5x + C_4\sin 5x\right)$

9. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = x(Ax + B) + xe^{-3x}(Cx + D)$ ________________ (correct)
(b) $y_p = Ax + B + e^{-3x}(Cx + D)$
(c) $y_p = x^2(Ax + B) + xe^{-3x}(Cx + D)$
(d) $y_p = x(Ax + B) + e^{-3x}(Cx + D)$
(e) $y_p = x(Ax + B) + x^2e^{-3x}(Cx + D)$

10. If $y_p = Axe^{3x} + Be^{3x} + C$ is a particular solution of the differential equation $y'' + 9y = 2xe^{3x} + 5$, then $9A + 27B + 9C =$

(a) 5 ________________ (correct)
(b) -5
(c) 1
(d) 9
(e) 0

11. A particular solution of the differential equation $y'' + y' = -e^{-2x}\cos(e^{-x})$ is given by $y_p(x) =$

(a) $\cos(e^{-x})$ ______________________________ (correct)

(b) $\sin(e^{-x})$

(c) $\cos(e^{-x}) - e^{-x}\sin(e^{-x})$

(d) $\cos(e^{-x}) + e^{-x}\sin(e^{-x})$

(e) $e^{-x}\cos(e^{-x}) + \sin(e^{-x})$

12. The characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ is

(a) $-\lambda^3 + 4\lambda^2 - 8\lambda + 8 = 0$ ______________________________ (correct)

(b) $-\lambda^3 + 4\lambda^2 + 8\lambda + 8 = 0$

(c) $-\lambda^3 - 4\lambda^2 - 8\lambda + 8 = 0$

(d) $\lambda^3 + 4\lambda^2 + 8\lambda - 8 = 0$

(e) $-\lambda^3 + 4\lambda^2 - 6\lambda + 8 = 0$

13. An eigenvector associated with the eigenvalue $\lambda = 5$ of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$ is $\begin{bmatrix} a \\ 2 \end{bmatrix}$, where $a =$

(a) 1 ____ (correct)
(b) -1
(c) 2
(d) -2
(e) 0

14. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 3)^2(\lambda + 2)$, then a basis for the eigenspace of $\lambda = 3$ is $v_1 = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$, and $v_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$, where $\alpha + \beta =$

(a) 6 ____ (correct)
(b) -6
(c) 5
(d) -5
(e) 0

15. If the matrix $A = \begin{bmatrix} 6 & -2 \\ 3 & 1 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$ ——————————(correct)

(b) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 1 **CODE 1**

MATH 208
Major Exam II
Term 253
19 July 2026
Net Time Allowed: 90 Minutes

Name			
ID		**Sec**	

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $y_p = Axe^{3x} + Be^{3x} + C$ is a particular solution of the differential equation $y'' + 9y = 2xe^{3x} + 5$, then $9A + 27B + 9C =$

   (a) 9
   (b) 0
   (c) 1
   (d) -5
   (e) 5

2. Given that $y = e^{4x}\cosh x$ is a solution of the differential equation

$$y^{(4)} - 12y^{(3)} + 60y'' - 164y' + 195y = 0,$$

   the general solution of the differential equation is

   (a) $y = C_1e^{5x} + C_2e^{3x} + e^{-2x}(C_3\cos 3x + C_4\sin 3x)$
   (b) $y = C_1e^{5x} + C_2e^{3x} + e^{x}(C_3\cos 5x + C_4\sin 5x)$
   (c) $y = C_1e^{-3x} + C_2e^{-5x} + e^{2x}(C_3\cos 4x + C_4\sin 4x)$
   (d) $y = C_1e^{5x} + C_2e^{3x} + e^{2x}(C_3\cos 3x + C_4\sin 3x)$
   (e) $y = C_1e^{-3x} + C_2e^{-5x} + e^{3x}(C_3\cos 2x + C_4\sin 2x)$

3. If the matrix $A = \begin{bmatrix} 6 & -2 \\ 3 & 1 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

4. The characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ is

(a) $-\lambda^3 + 4\lambda^2 - 6\lambda + 8 = 0$

(b) $-\lambda^3 + 4\lambda^2 - 8\lambda + 8 = 0$

(c) $-\lambda^3 + 4\lambda^2 + 8\lambda + 8 = 0$

(d) $-\lambda^3 - 4\lambda^2 - 8\lambda + 8 = 0$

(e) $\lambda^3 + 4\lambda^2 + 8\lambda - 8 = 0$

5. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) \in \mathbb{R}^4 \mid a+2b+3c+4d = 0,\ b+2c+3d = 0\}$. A basis of S consists of the vectors

   (a) $v_1 = (1, 2, 1, 0),\ v_2 = (2, -3, 0, 1)$
   (b) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$
   (c) $v_1 = (2, -3, 0, 1),\ v_2 = (4, -6, 0, 2)$
   (d) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -4, 2, 0)$
   (e) $v_1 = (-1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$

6. The rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$ is

   (a) 0
   (b) 3
   (c) 2
   (d) 1
   (e) 4

7. If the solution space of the system

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0, \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0, \\ x_1 + 3x_2 + 3x_3 + 13x_4 &= 0 \end{aligned}$$

consists of all linear combinations of the vectors $v_1 = (a, b, 1, 0)$ and $v_2 = (c, d, 0, 1)$, then $a+b+c+d =$

(a) -4
(b) 6
(c) 4
(d) -6
(e) 0

8. An eigenvector associated with the eigenvalue $\lambda = 5$ of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$ is $\begin{bmatrix} a \\ 2 \end{bmatrix}$, where $a =$

(a) 0
(b) -2
(c) 2
(d) -1
(e) 1

9. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 3)^2(\lambda + 2)$, then a basis for the eigenspace of $\lambda = 3$ is $v_1 = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$, and $v_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$, where $\alpha + \beta =$

   (a) -6
   (b) 6
   (c) 5
   (d) -5
   (e) 0

10. The general solution of the differential equation $y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$ is

   (a) $y(x) = \big(C_1 + C_2 x\big)e^{x} + \big(C_3 + C_4 x\big)e^{-2x}$
   (b) $y(x) = \big(C_1 + C_2 x + C_3 x^2\big)e^{x} + C_4 x e^{-2x}$
   (c) $y(x) = C_1 e^{x} + \big(C_2 + C_3 x + C_4 x^2\big)e^{-2x}$
   (d) $y(x) = \big(C_1 + C_2 x\big)e^{x} + C_3 e^{-x} + C_4 e^{-2x}$
   (e) $y(x) = \big(C_1 + C_2 x + C_3 x^2\big)e^{x} + C_4 e^{-2x}$

11. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = x(Ax + B) + x^2 e^{-3x}(Cx + D)$
(b) $y_p = x(Ax + B) + xe^{-3x}(Cx + D)$
(c) $y_p = Ax + B + e^{-3x}(Cx + D)$
(d) $y_p = x^2(Ax + B) + xe^{-3x}(Cx + D)$
(e) $y_p = x(Ax + B) + e^{-3x}(Cx + D)$

12. A particular solution of the differential equation $y'' + y' = -e^{-2x}\cos(e^{-x})$ is given by $y_p(x) =$

(a) $\cos(e^{-x}) + e^{-x}\sin(e^{-x})$
(b) $e^{-x}\cos(e^{-x}) + \sin(e^{-x})$
(c) $\cos(e^{-x}) - e^{-x}\sin(e^{-x})$
(d) $\cos(e^{-x})$
(e) $\sin(e^{-x})$

13. If $y(x)$ is the solution of the initial-value problem

$$35y'' - y' - 12y = 0, \qquad y(0) = 5, \quad y'(0) = 3,$$

then $y(1) =$

(a) $5e^{3/5}$
(b) 0
(c) $3e^{3/5} + 2e^{-4/7}$
(d) $2e^{3/5} + 3e^{-4/7}$
(e) $5e^{-4/7}$

14. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx)\cos 2x + (C + Dx)\sin 2x + Ee^{2x} + Fe^{-2x}$$

is

(a) $y^{(6)} + 4y^{(4)} - 16y'' - 64y = 0$
(b) $y^{(6)} + 8y^{(4)} + 16y'' = 0$
(c) $y^{(4)} - 16y = 0$
(d) $y^{(6)} - 4y^{(4)} - 16y'' + 64y = 0$
(e) $y^{(6)} - 8y^{(4)} + 16y'' = 0$

15. If $W(x)$ is the Wronskian of the functions

$$f_1(x) = \sin(5x) + 2\cos(5x), \text{ and } f_2(x) = 3\sin(5x) + \cos(5x),$$

then $W(x) =$

(a) 5
(b) 0
(c) 25
(d) -25
(e) $25\cos(5x)$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 2 **CODE 2**

MATH 208
Major Exam II
Term 253
19 July 2026
Net Time Allowed: 90 Minutes

Name			
ID		**Sec**	

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$ is

(a) 0
(b) 3
(c) 2
(d) 4
(e) 1

2. A particular solution of the differential equation $y'' + y' = -e^{-2x}\cos(e^{-x})$ is given by $y_p(x) =$

(a) $\cos(e^{-x})$
(b) $\cos(e^{-x}) + e^{-x}\sin(e^{-x})$
(c) $\sin(e^{-x})$
(d) $\cos(e^{-x}) - e^{-x}\sin(e^{-x})$
(e) $e^{-x}\cos(e^{-x}) + \sin(e^{-x})$

3. An eigenvector associated with the eigenvalue $\lambda = 5$ of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$ is $\begin{bmatrix} a \\ 2 \end{bmatrix}$, where $a =$

   (a) 0
   (b) -1
   (c) 2
   (d) 1
   (e) -2

4. If $y(x)$ is the solution of the initial-value problem

$$35y'' - y' - 12y = 0, \qquad y(0) = 5, \quad y'(0) = 3,$$

   then $y(1) =$

   (a) $5e^{3/5}$
   (b) $2e^{3/5} + 3e^{-4/7}$
   (c) $5e^{-4/7}$
   (d) 0
   (e) $3e^{3/5} + 2e^{-4/7}$

5. The general solution of the differential equation $y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$ is

   (a) $y(x) = \big(C_1 + C_2x + C_3x^2\big)e^{x} + C_4xe^{-2x}$
   (b) $y(x) = C_1e^{x} + \big(C_2 + C_3x + C_4x^2\big)e^{-2x}$
   (c) $y(x) = \big(C_1 + C_2x\big)e^{x} + \big(C_3 + C_4x\big)e^{-2x}$
   (d) $y(x) = \big(C_1 + C_2x\big)e^{x} + C_3e^{-x} + C_4e^{-2x}$
   (e) $y(x) = \big(C_1 + C_2x + C_3x^2\big)e^{x} + C_4e^{-2x}$

6. Given that $y = e^{4x}\cosh x$ is a solution of the differential equation

   $$y^{(4)} - 12y^{(3)} + 60y'' - 164y' + 195y = 0,$$

   the general solution of the differential equation is

   (a) $y = C_1e^{-3x} + C_2e^{-5x} + e^{2x}\left(C_3\cos 4x + C_4\sin 4x\right)$
   (b) $y = C_1e^{5x} + C_2e^{3x} + e^{x}\left(C_3\cos 5x + C_4\sin 5x\right)$
   (c) $y = C_1e^{-3x} + C_2e^{-5x} + e^{3x}\left(C_3\cos 2x + C_4\sin 2x\right)$
   (d) $y = C_1e^{5x} + C_2e^{3x} + e^{2x}\left(C_3\cos 3x + C_4\sin 3x\right)$
   (e) $y = C_1e^{5x} + C_2e^{3x} + e^{-2x}\left(C_3\cos 3x + C_4\sin 3x\right)$

7. If $y_p = Axe^{3x} + Be^{3x} + C$ is a particular solution of the differential equation $y'' + 9y = 2xe^{3x} + 5$, then $9A + 27B + 9C =$

(a) -5

(b) 1

(c) 0

(d) 5

(e) 9

8. If the matrix $A = \begin{bmatrix} 6 & -2 \\ 3 & 1 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

(e) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

9. The characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ is

(a) $-\lambda^3 + 4\lambda^2 - 8\lambda + 8 = 0$
(b) $-\lambda^3 - 4\lambda^2 - 8\lambda + 8 = 0$
(c) $\lambda^3 + 4\lambda^2 + 8\lambda - 8 = 0$
(d) $-\lambda^3 + 4\lambda^2 + 8\lambda + 8 = 0$
(e) $-\lambda^3 + 4\lambda^2 - 6\lambda + 8 = 0$

10. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = x^2(Ax + B) + xe^{-3x}(Cx + D)$
(b) $y_p = x(Ax + B) + xe^{-3x}(Cx + D)$
(c) $y_p = x(Ax + B) + e^{-3x}(Cx + D)$
(d) $y_p = x(Ax + B) + x^2e^{-3x}(Cx + D)$
(e) $y_p = Ax + B + e^{-3x}(Cx + D)$

11. If $W(x)$ is the Wronskian of the functions

$$f_1(x) = \sin(5x) + 2\cos(5x), \text{ and } f_2(x) = 3\sin(5x) + \cos(5x),$$

then $W(x) =$

(a) 0
(b) -25
(c) 25
(d) 5
(e) $25\cos(5x)$

12. If the solution space of the system

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0, \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0, \\ x_1 + 3x_2 + 3x_3 + 13x_4 &= 0 \end{aligned}$$

consists of all linear combinations of the vectors $v_1 = (a, b, 1, 0)$ and $v_2 = (c, d, 0, 1)$, then $a+b+c+d =$

(a) 4
(b) -4
(c) 6
(d) 0
(e) -6

13. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 3)^2(\lambda + 2)$, then a basis for the eigenspace of $\lambda = 3$ is $v_1 = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$, and $v_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$, where $\alpha + \beta =$

(a) 5
(b) 6
(c) 0
(d) -5
(e) -6

14. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx)\cos 2x + (C + Dx)\sin 2x + Ee^{2x} + Fe^{-2x}$$

is

(a) $y^{(6)} - 4y^{(4)} - 16y'' + 64y = 0$
(b) $y^{(6)} + 8y^{(4)} + 16y'' = 0$
(c) $y^{(4)} - 16y = 0$
(d) $y^{(6)} + 4y^{(4)} - 16y'' - 64y = 0$
(e) $y^{(6)} - 8y^{(4)} + 16y'' = 0$

15. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a,b,c,d) \in \mathbb{R}^4 \mid a+2b+3c+4d = 0,\ b+2c+3d = 0\}$. A basis of S consists of the vectors

(a) $v_1 = (-1,-2,1,0),\ v_2 = (2,-3,0,1)$

(b) $v_1 = (1,-2,1,0),\ v_2 = (2,-4,2,0)$

(c) $v_1 = (1,-2,1,0),\ v_2 = (2,-3,0,1)$

(d) $v_1 = (2,-3,0,1),\ v_2 = (4,-6,0,2)$

(e) $v_1 = (1,2,1,0),\ v_2 = (2,-3,0,1)$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 3 **CODE 3**

MATH 208
Major Exam II
Term 253
19 July 2026
Net Time Allowed: 90 Minutes

Name	
ID		Sec	

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ is

   (a) $-\lambda^3 + 4\lambda^2 + 8\lambda + 8 = 0$
   (b) $\lambda^3 + 4\lambda^2 + 8\lambda - 8 = 0$
   (c) $-\lambda^3 + 4\lambda^2 - 6\lambda + 8 = 0$
   (d) $-\lambda^3 + 4\lambda^2 - 8\lambda + 8 = 0$
   (e) $-\lambda^3 - 4\lambda^2 - 8\lambda + 8 = 0$

2. If the solution space of the system

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0, \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0, \\ x_1 + 3x_2 + 3x_3 + 13x_4 &= 0 \end{aligned}$$

   consists of all linear combinations of the vectors $v_1 = (a, b, 1, 0)$ and $v_2 = (c, d, 0, 1)$, then $a+b+c+d =$

   (a) 0
   (b) 6
   (c) 4
   (d) -4
   (e) -6

3. If the matrix $A = \begin{bmatrix} 6 & -2 \\ 3 & 1 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $\quad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(b) $P = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, $\quad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$, $\quad D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

(d) $P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $\quad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(e) $P = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $\quad D = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$

4. The rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$ is

(a) 4

(b) 3

(c) 1

(d) 2

(e) 0

5. The general solution of the differential equation $y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$ is

   (a) $y(x) = C_1e^x + \big(C_2 + C_3x + C_4x^2\big)e^{-2x}$
   (b) $y(x) = \big(C_1 + C_2x\big)e^x + \big(C_3 + C_4x\big)e^{-2x}$
   (c) $y(x) = \big(C_1 + C_2x + C_3x^2\big)e^x + C_4xe^{-2x}$
   (d) $y(x) = \big(C_1 + C_2x\big)e^x + C_3e^{-x} + C_4e^{-2x}$
   (e) $y(x) = \big(C_1 + C_2x + C_3x^2\big)e^x + C_4e^{-2x}$

6. A particular solution of the differential equation $y'' + y' = -e^{-2x}\cos(e^{-x})$ is given by $y_p(x) =$

   (a) $\cos(e^{-x})$
   (b) $e^{-x}\cos(e^{-x}) + \sin(e^{-x})$
   (c) $\cos(e^{-x}) + e^{-x}\sin(e^{-x})$
   (d) $\sin(e^{-x})$
   (e) $\cos(e^{-x}) - e^{-x}\sin(e^{-x})$

7. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx)\cos 2x + (C + Dx)\sin 2x + Ee^{2x} + Fe^{-2x}$$

is

(a) $y^{(6)} - 4y^{(4)} - 16y'' + 64y = 0$
(b) $y^{(6)} + 4y^{(4)} - 16y'' - 64y = 0$
(c) $y^{(6)} - 8y^{(4)} + 16y'' = 0$
(d) $y^{(4)} - 16y = 0$
(e) $y^{(6)} + 8y^{(4)} + 16y'' = 0$

8. Given that $y = e^{4x}\cosh x$ is a solution of the differential equation

$$y^{(4)} - 12y^{(3)} + 60y'' - 164y' + 195y = 0,$$

the general solution of the differential equation is

(a) $y = C_1e^{5x} + C_2e^{3x} + e^{x}\,(C_3\cos 5x + C_4\sin 5x)$
(b) $y = C_1e^{-3x} + C_2e^{-5x} + e^{2x}\,(C_3\cos 4x + C_4\sin 4x)$
(c) $y = C_1e^{5x} + C_2e^{3x} + e^{-2x}\,(C_3\cos 3x + C_4\sin 3x)$
(d) $y = C_1e^{5x} + C_2e^{3x} + e^{2x}\,(C_3\cos 3x + C_4\sin 3x)$
(e) $y = C_1e^{-3x} + C_2e^{-5x} + e^{3x}\,(C_3\cos 2x + C_4\sin 2x)$

9. If $W(x)$ is the Wronskian of the functions

$$f_1(x) = \sin(5x) + 2\cos(5x), \text{ and } f_2(x) = 3\sin(5x) + \cos(5x),$$

then $W(x) =$

(a) $25\cos(5x)$
(b) 5
(c) 0
(d) -25
(e) 25

10. If $y_p = Axe^{3x} + Be^{3x} + C$ is a particular solution of the differential equation $y'' + 9y = 2xe^{3x} + 5$, then $9A + 27B + 9C =$

(a) -5
(b) 9
(c) 5
(d) 1
(e) 0

11. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) \in \mathbb{R}^4 \mid a+2b+3c+4d = 0,\ b+2c+3d = 0\}$. A basis of S consists of the vectors

(a) $v_1 = (2, -3, 0, 1),\ v_2 = (4, -6, 0, 2)$
(b) $v_1 = (-1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$
(c) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -4, 2, 0)$
(d) $v_1 = (1, 2, 1, 0),\ v_2 = (2, -3, 0, 1)$
(e) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$

12. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = x^2(Ax + B) + xe^{-3x}(Cx + D)$
(b) $y_p = x(Ax + B) + x^2e^{-3x}(Cx + D)$
(c) $y_p = x(Ax + B) + xe^{-3x}(Cx + D)$
(d) $y_p = Ax + B + e^{-3x}(Cx + D)$
(e) $y_p = x(Ax + B) + e^{-3x}(Cx + D)$

13. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ is $p(\lambda) = -(\lambda - 3)^2(\lambda + 2)$, then a basis for the eigenspace of $\lambda = 3$ is $v_1 = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$, and $v_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$, where $\alpha + \beta =$

(a) -6
(b) 6
(c) 5
(d) -5
(e) 0

14. If $y(x)$ is the solution of the initial-value problem

$$35y'' - y' - 12y = 0, \qquad y(0) = 5, \quad y'(0) = 3,$$

then $y(1) =$

(a) $5e^{3/5}$
(b) $2e^{3/5} + 3e^{-4/7}$
(c) 0
(d) $5e^{-4/7}$
(e) $3e^{3/5} + 2e^{-4/7}$

15. An eigenvector associated with the eigenvalue $\lambda = 5$ of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$ is $\begin{bmatrix} a \\ 2 \end{bmatrix}$, where $a =$

(a) -2
(b) -1
(c) 1
(d) 0
(e) 2

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 4 **CODE 4**

MATH 208
Major Exam II
Term 253
19 July 2026
Net Time Allowed: 90 Minutes

Name			
ID		**Sec**	

Check that this exam has 15 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. If $y_p = Axe^{3x} + Be^{3x} + C$ is a particular solution of the differential equation $y'' + 9y = 2xe^{3x} + 5$, then $9A + 27B + 9C =$

   (a) 5
   (b) -5
   (c) 1
   (d) 9
   (e) 0

2. A particular solution of the differential equation $y'' + y' = -e^{-2x}\cos(e^{-x})$ is given by $y_p(x) =$

   (a) $e^{-x}\cos(e^{-x}) + \sin(e^{-x})$
   (b) $\cos(e^{-x})$
   (c) $\sin(e^{-x})$
   (d) $\cos(e^{-x}) - e^{-x}\sin(e^{-x})$
   (e) $\cos(e^{-x}) + e^{-x}\sin(e^{-x})$

3. Consider the subspace S of $\mathbb{R}^4$ defined by $S = \{(a, b, c, d) \in \mathbb{R}^4 \mid a+2b+3c+4d = 0,\ b+2c+3d = 0\}$. A basis of S consists of the vectors

   (a) $v_1 = (-1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$
   (b) $v_1 = (2, -3, 0, 1),\ v_2 = (4, -6, 0, 2)$
   (c) $v_1 = (1, 2, 1, 0),\ v_2 = (2, -3, 0, 1)$
   (d) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -3, 0, 1)$
   (e) $v_1 = (1, -2, 1, 0),\ v_2 = (2, -4, 2, 0)$

4. If the solution space of the system

$$\begin{aligned} x_1 - 3x_2 - 9x_3 - 5x_4 &= 0, \\ 2x_1 + x_2 - 4x_3 + 11x_4 &= 0, \\ x_1 + 3x_2 + 3x_3 + 13x_4 &= 0 \end{aligned}$$

   consists of all linear combinations of the vectors $v_1 = (a, b, 1, 0)$ and $v_2 = (c, d, 0, 1)$, then $a+b+c+d =$

   (a) 4
   (b) 0
   (c) 6
   (d) -4
   (e) -6

5. If $y(x)$ is the solution of the initial-value problem

$$35y'' - y' - 12y = 0, \qquad y(0) = 5, \quad y'(0) = 3,$$

then $y(1) =$

(a) 0
(b) $5e^{3/5}$
(c) $2e^{3/5} + 3e^{-4/7}$
(d) $5e^{-4/7}$
(e) $3e^{3/5} + 2e^{-4/7}$

6. A linear homogeneous constant-coefficient differential equation which has the general solution

$$y(x) = (A + Bx)\cos 2x + (C + Dx)\sin 2x + Ee^{2x} + Fe^{-2x}$$

is

(a) $y^{(6)} + 8y^{(4)} + 16y'' = 0$
(b) $y^{(6)} - 4y^{(4)} - 16y'' + 64y = 0$
(c) $y^{(6)} - 8y^{(4)} + 16y'' = 0$
(d) $y^{(6)} + 4y^{(4)} - 16y'' - 64y = 0$
(e) $y^{(4)} - 16y = 0$

7. Given that $y = e^{4x}\cosh x$ is a solution of the differential equation

$$y^{(4)} - 12y^{(3)} + 60y'' - 164y' + 195y = 0,$$

the general solution of the differential equation is

(a) $y = C_1e^{-3x} + C_2e^{-5x} + e^{2x}\left(C_3\cos 4x + C_4\sin 4x\right)$
(b) $y = C_1e^{5x} + C_2e^{3x} + e^{-2x}\left(C_3\cos 3x + C_4\sin 3x\right)$
(c) $y = C_1e^{5x} + C_2e^{3x} + e^{x}\left(C_3\cos 5x + C_4\sin 5x\right)$
(d) $y = C_1e^{-3x} + C_2e^{-5x} + e^{3x}\left(C_3\cos 2x + C_4\sin 2x\right)$
(e) $y = C_1e^{5x} + C_2e^{3x} + e^{2x}\left(C_3\cos 3x + C_4\sin 3x\right)$

8. The characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ is

(a) $-\lambda^3 + 4\lambda^2 + 8\lambda + 8 = 0$
(b) $-\lambda^3 + 4\lambda^2 - 8\lambda + 8 = 0$
(c) $-\lambda^3 + 4\lambda^2 - 6\lambda + 8 = 0$
(d) $-\lambda^3 - 4\lambda^2 - 8\lambda + 8 = 0$
(e) $\lambda^3 + 4\lambda^2 + 8\lambda - 8 = 0$

9. An eigenvector associated with the eigenvalue $\lambda = 5$ of the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 3 \end{bmatrix}$ is $\begin{bmatrix} a \\ 2 \end{bmatrix}$, where $a =$

(a) 1
(b) 0
(c) -1
(d) 2
(e) -2

10. The general solution of the differential equation $y^{(4)} - y^{(3)} - 3y'' + 5y' - 2y = 0$ is

(a) $y(x) = \big(C_1 + C_2x + C_3x^2\big)e^x + C_4e^{-2x}$
(b) $y(x) = \big(C_1 + C_2x\big)e^x + C_3e^{-x} + C_4e^{-2x}$
(c) $y(x) = C_1e^x + \big(C_2 + C_3x + C_4x^2\big)e^{-2x}$
(d) $y(x) = \big(C_1 + C_2x + C_3x^2\big)e^x + C_4xe^{-2x}$
(e) $y(x) = \big(C_1 + C_2x\big)e^x + \big(C_3 + C_4x\big)e^{-2x}$

11. If the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ is $p(\lambda) = -(\lambda-3)^2(\lambda+2)$, then a basis for the eigenspace of $\lambda = 3$ is $v_1 = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$, and $v_2 = \begin{bmatrix} \beta \\ 0 \\ 1 \end{bmatrix}$, where $\alpha + \beta =$

(a) -6
(b) 0
(c) 5
(d) -5
(e) 6

12. The rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 1 & 1 & 3 \\ 2 & 1 & 3 & 4 \end{bmatrix}$ is

(a) 3
(b) 0
(c) 2
(d) 4
(e) 1

13. If $W(x)$ is the Wronskian of the functions

$$f_1(x) = \sin(5x) + 2\cos(5x), \text{ and } f_2(x) = 3\sin(5x) + \cos(5x),$$

then $W(x) =$

(a) 5
(b) 25
(c) -25
(d) $25\cos(5x)$
(e) 0

14. If the matrix $A = \begin{bmatrix} 6 & -2 \\ 3 & 1 \end{bmatrix}$ is diagonalizable with a diagonalizing matrix P and a diagonal matrix D such that $P^{-1}AP = D$, then

(a) $P = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$
(b) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$
(c) $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$
(d) $P = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}, \qquad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$
(e) $P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

15. An appropriate form of a particular solution y_p for the non-homogeneous differential equation

$$y^{(3)} - y'' - 12y' = x - 2xe^{-3x}$$

is

(a) $y_p = x(Ax + B) + x^2e^{-3x}(Cx + D)$
(b) $y_p = x(Ax + B) + e^{-3x}(Cx + D)$
(c) $y_p = Ax + B + e^{-3x}(Cx + D)$
(d) $y_p = x(Ax + B) + xe^{-3x}(Cx + D)$
(e) $y_p = x^2(Ax + B) + xe^{-3x}(Cx + D)$

MATH 208, Term 253, Major Exam II – Answer Key

Q	MASTER	1	2	3	4
1	A	E 10	C 3	D 12	A 10
2	A	D 8	A 11	E 1	B 11
3	A	C 15	D 13	A 15	D 2
4	A	B 12	A 5	D 3	E 1
5	A	B 2	E 6	E 6	B 5
6	A	C 3	D 8	A 11	D 7
7	A	D 1	D 10	B 7	E 8
8	A	E 13	E 15	D 8	B 12
9	A	B 14	A 12	E 4	A 13
10	A	E 6	B 9	C 10	A 6
11	A	B 9	C 4	E 2	E 14
12	A	D 11	E 1	C 9	C 3
13	A	A 5	B 14	B 14	B 4
14	A	A 7	D 7	A 5	C 15
15	A	C 4	C 2	C 13	D 9

Answer Counts

V	A	B	C	D	E
1	2	4	3	3	3
2	3	2	3	4	3
3	3	2	3	3	4
4	3	4	2	3	3

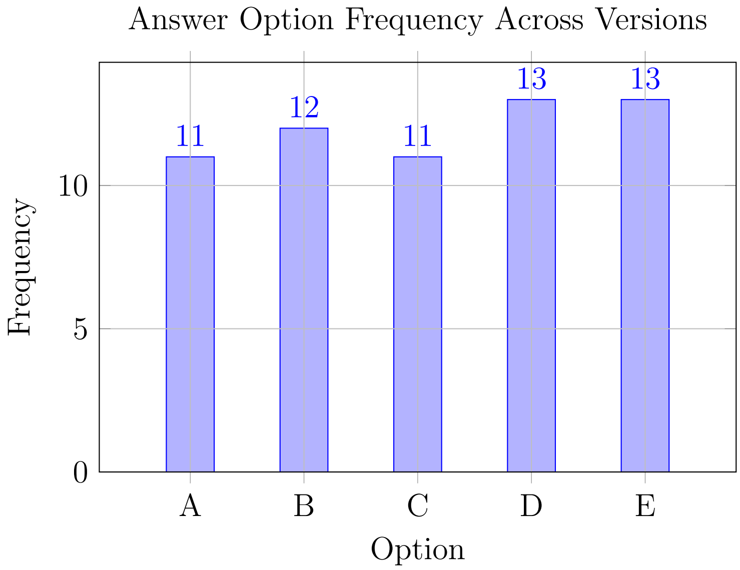

Figure 1: Frequency of each answer option, excluding the master.