King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 208
Final Exam
Term 253
August 5, 2026

EXAM COVER

Number of versions: 4
Number of questions: 21

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Department of Mathematics

MATH 208
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Net Time Allowed: 120 Minutes

MASTER VERSION

1. The solution of the exact differential equation

$$\left(\sin y + y\sin x + \frac{1}{x}\right)\, dx + \left(\frac{1}{y} + x\cos y - \cos x\right)\, dy = 0$$

is given by

(a) $\ln|x| + \ln|y| + x\sin y - y\cos x = C$ ____________________ (correct)

(b) $\ln|x| + \ln|y| + x\cos y - y\cos x = C$

(c) $\ln|x| - \ln|y| - x\cos y - y\cos x = C$

(d) $\ln|x| - \ln|y| + x\sin y - y\cos x = C$

(e) $\ln|x| + \ln|y| + y\sin y - x\cos x = C$

2. A general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2}{x}y = 4x^3y^4$ is given by

(a) $y^{-3} = 6x^4 + Cx^6$ ____________________ (correct)

(b) $y^{-3} = 2x^4 + Cx^6$

(c) $y^{-3} = 4x^4 + Cx^6$

(d) $y^{-3} = 3x^4 + Cx^6$

(e) $y^{-3} = 5x^4 + Cx^6$

3. The solution of the initial-value problem

$$xe^y\,dy + \left(\frac{x^2+1}{y}\right)dx = 0, \qquad y(1) = 1,$$

is given by

(a) $2(y-1)e^y + x^2 + 2\ln|x| = 1$ ________________ (correct)
(b) $3(y-1)e^y + x^2 + 2\ln|x| = 1$
(c) $4(y-1)e^y + x^2 + 3\ln|x| = 1$
(d) $5(y-1)e^y + x^2 - 2\ln|x| = 1$
(e) $(y-1)e^y + x^2 + 2\ln|x| = 1$

4. The sum of the indicial roots at $x = 0$ for the differential equation

$$x(4-x)y'' + (2-x)y' + 4y = 0$$

is

(a) $\frac{1}{2}$ ________________ (correct)
(b) 0
(c) $-\frac{1}{2}$
(d) $\frac{1}{4}$
(e) $-\frac{1}{4}$

5. If the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & -3 & 5 & 4 \\ 3 & -5 & 8 & 5 \\ 1 & -1 & 2 & k^2-6 \end{bmatrix}$ is equal to 2, then $k =$

   (a) ± 3 ____ (correct)
   (b) 3 only
   (c) -3 only
   (d) $\pm\sqrt{6}$
   (e) 0

6. Suppose that a 2×2 matrix A has the eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$, with corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, respectively. If $A^2 = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, then $x + y + z + w =$
   (Hint: Find a diagonalizing matrix P and write $A = PDP^{-1}$.)

   (a) -7 ____ (correct)
   (b) -6
   (c) -5
   (d) -9
   (e) -10

7. The number of irregular singular points of the differential equation

$$x^2(1-x^2)y'' + y' + xy = 0$$

is

(a) 1 ____ (correct)

(b) 2

(c) 3

(d) 0

(e) 4

8. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power-series solution of the differential equation $(x^2+1)y'' - 4xy' + 6y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation

(a) $c_n = \dfrac{(4-n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$ ____ (correct)

(b) $c_n = \dfrac{(4+n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(c) $c_n = \dfrac{(4-n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(d) $c_n = \dfrac{(4+n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(e) $c_n = \dfrac{(2+n)(n-5)}{n(n+1)} c_{n-2}, \quad n \geq 2$

9. Consider the system

$$X' = AX, \qquad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

where A is a 2×2 matrix with real entries. If A has the eigenvalue $\lambda = 1 + i$ with corresponding eigenvector $K = \begin{bmatrix} -1-i \\ 1 \end{bmatrix}$, then $X(\pi) =$

(a) $e^{\pi} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ ____(correct)

(b) $e^{\pi} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $e^{\pi} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $e^{\pi} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(e) $e^{\pi} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

10. If $K = \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = -2$ of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & -3 \\ 1 & -4 & 2 \end{bmatrix}$,
then $\alpha =$

(a) 0 ____(correct)

(b) -1

(c) 1

(d) 2

(e) -2

11. A fundamental matrix of the system $X' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} -e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$ ——————————————————(correct)

(b) $\begin{bmatrix} e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} 2e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{-t} & 3e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{bmatrix}$

12. If the three vectors $v_1 = (1, 2, 3)$, $v_2 = (-1, 0, 4)$, and $v_3 = (3, 2, k)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) -5 ——————————————————(correct)

(b) $-\frac{2}{3}$

(c) $\frac{5}{2}$

(d) -2

(e) 7

13. Consider the non-homogeneous system

$$X' = AX + \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If the general solution of the associated homogeneous system is $X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t}$, then a particular solution X_p is

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ ________________ (correct)

(b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(e) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

14. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' - 2y' = 6e^{2x} + 5\cos x$$

is given by $y_p = Axe^{2x} + B\cos x + C\sin x$. Then $5B + 2A + 5C =$

(a) -9 ________________ (correct)

(b) 5

(c) -3

(d) 7

(e) 0

15. A particular solution of the differential equation $y'' + 4y = 4\sec(2x)$ is given by $y_p(x) =$

(a) $\cos(2x)\ln|\cos(2x)| + 2x\sin(2x)$ ______________________________ (correct)

(b) $-\cos(2x)\ln|\sin(2x)| + x\sin(2x)$

(c) $\sin(2x)\ln|\sec(2x) + \tan(2x)| - 1$

(d) $\cos(2x)\ln|\sec(2x) + \tan(2x)| + x\sin(2x)$

(e) $2x\cos(2x) + \sin(2x)\ln|\cos(2x)|$

16. The minimum radius of convergence of a power-series solution of the differential equation

$$(x^2 - 4x + 13)y'' + 2xy' - y = 0$$

about the ordinary point $x = 1$ is equal to

(a) $\sqrt{10}$ ______________________________ (correct)

(b) 3

(c) $\sqrt{13}$

(d) $2\sqrt{13}$

(e) 5

17. If $c_0 = 1$, $c_k = \dfrac{2k-7}{12k} c_{k-1}, k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of a differential equation about the regular singular point $x = 0$, then the solution is given by

(a) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$ ——————————(correct)

(b) $y = x^{1/2}\left[1 + \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$

(c) $y = x^{1/2}\left[1 - \dfrac{5}{12}x - \dfrac{5}{96}x^2 + \cdots\right]$

(d) $y = x^{1/2}\left[1 - \dfrac{5}{18}x + \dfrac{5}{96}x^2 + \cdots\right]$

(e) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{7}{96}x^2 + \cdots\right]$

18. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$ ——————————(correct)

(b) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & e^{5t} \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & 2te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

19. The general solution of $X' = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ -2 \\ 1 \end{bmatrix} e^{t}.$$

Then $\alpha + \beta + \gamma =$

(a) 1 ____ (correct)
(b) 0
(c) -2
(d) 3
(e) -1

20. A linear homogeneous differential equation with real constant coefficients having the solutions xe^{2x} and $3e^{2x}\sin(2x)$ is given by

(a) $y^{(4)} - 8y^{(3)} + 28y'' - 48y' + 32y = 0$ ________________________ (correct)
(b) $y^{(4)} - 8y^{(3)} + 29y'' - 52y' + 40y = 0$
(c) $y^{(4)} + 8y^{(3)} + 29y'' + 52y' + 40y = 0$
(d) $y^{(4)} - 6y^{(3)} + 20y'' - 40y' + 32y = 0$
(e) $y^{(4)} - 8y^{(3)} + 29y'' - 40y' + 16y = 0$

21. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, which has an eigenvalue $\lambda = 2$ of multiplicity 3 and defect 2. If we choose the generalized eigenvector $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ such that $(A - 2I)^3 v_3 = 0$ but $(A - 2I)^2 v_3 \neq 0$, which of the following represents the general solution of the differential system $X' = AX$?

(a) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2/2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$ ________________ (correct)

(b) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t+1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ 2t \\ 1 \end{bmatrix} \right) e^{2t}$

(c) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t \\ t^2/2 \\ 1 \end{bmatrix} \right) e^{2t}$

(d) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(e) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t \\ t^2/2 \end{bmatrix} \right) e^{2t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 5 **CODE 5**

MATH 208
Final Exam
Term 253
August 5, 2026
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. Consider the non-homogeneous system

$$X' = AX + \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If the general solution of the associated homogeneous system is $X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t}$, then a particular solution X_p is

(a) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

2. Consider the system

$$X' = AX, \qquad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

where A is a 2×2 matrix with real entries. If A has the eigenvalue $\lambda = 1 + i$ with corresponding eigenvector $K = \begin{bmatrix} -1 - i \\ 1 \end{bmatrix}$, then $X(\pi) =$

(a) $e^{\pi} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(b) $e^{\pi} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

(c) $e^{\pi} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

(d) $e^{\pi} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(e) $e^{\pi} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

3. The solution of the exact differential equation

$$\left(\sin y + y\sin x + \frac{1}{x}\right)\,dx + \left(\frac{1}{y} + x\cos y - \cos x\right)\,dy = 0$$

is given by

(a) $\ln|x| - \ln|y| - x\cos y - y\cos x = C$
(b) $\ln|x| - \ln|y| + x\sin y - y\cos x = C$
(c) $\ln|x| + \ln|y| + x\cos y - y\cos x = C$
(d) $\ln|x| + \ln|y| + y\sin y - x\cos x = C$
(e) $\ln|x| + \ln|y| + x\sin y - y\cos x = C$

4. The general solution of $X' = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ -2 \\ 1 \end{bmatrix} e^{t}.$$

Then $\alpha + \beta + \gamma =$

(a) -1
(b) 3
(c) 1
(d) 0
(e) -2

5. If the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & -3 & 5 & 4 \\ 3 & -5 & 8 & 5 \\ 1 & -1 & 2 & k^2-6 \end{bmatrix}$ is equal to 2, then $k =$

(a) -3 only
(b) ± 3
(c) $\pm\sqrt{6}$
(d) 3 only
(e) 0

6. A general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2}{x}y = 4x^3y^4$ is given by

(a) $y^{-3} = 3x^4 + Cx^6$
(b) $y^{-3} = 6x^4 + Cx^6$
(c) $y^{-3} = 4x^4 + Cx^6$
(d) $y^{-3} = 2x^4 + Cx^6$
(e) $y^{-3} = 5x^4 + Cx^6$

7. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power-series solution of the differential equation $(x^2+1)y'' - 4xy' + 6y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation

(a) $c_n = \dfrac{(4+n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(b) $c_n = \dfrac{(4-n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(c) $c_n = \dfrac{(4-n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(d) $c_n = \dfrac{(2+n)(n-5)}{n(n+1)} c_{n-2}, \quad n \geq 2$

(e) $c_n = \dfrac{(4+n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

8. The solution of the initial-value problem

$$xe^y\,dy + \left(\frac{x^2+1}{y}\right)dx = 0, \qquad y(1) = 1,$$

is given by

(a) $3(y-1)e^y + x^2 + 2\ln|x| = 1$

(b) $2(y-1)e^y + x^2 + 2\ln|x| = 1$

(c) $5(y-1)e^y + x^2 - 2\ln|x| = 1$

(d) $(y-1)e^y + x^2 + 2\ln|x| = 1$

(e) $4(y-1)e^y + x^2 + 3\ln|x| = 1$

9. Suppose that a 2×2 matrix A has the eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$, with corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, respectively. If $A^2 = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, then $x + y + z + w =$
(Hint: Find a diagonalizing matrix P and write $A = PDP^{-1}$.)

(a) -7
(b) -10
(c) -9
(d) -5
(e) -6

10. The minimum radius of convergence of a power-series solution of the differential equation

$$(x^2 - 4x + 13)y'' + 2xy' - y = 0$$

about the ordinary point $x = 1$ is equal to

(a) $\sqrt{10}$
(b) $2\sqrt{13}$
(c) $\sqrt{13}$
(d) 3
(e) 5

11. If $K = \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = -2$ of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & -3 \\ 1 & -4 & 2 \end{bmatrix}$, then $\alpha =$

    (a) 0
    (b) -1
    (c) -2
    (d) 2
    (e) 1

12. The number of irregular singular points of the differential equation

$$x^2(1-x^2)y'' + y' + xy = 0$$

is

    (a) 0
    (b) 2
    (c) 3
    (d) 1
    (e) 4

13. A fundamental matrix of the system $X' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} -e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} 2e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{-t} & 3e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

14. A linear homogeneous differential equation with real constant coefficients having the solutions xe^{2x} and $3e^{2x}\sin(2x)$ is given by

(a) $y^{(4)} - 8y^{(3)} + 29y'' - 52y' + 40y = 0$

(b) $y^{(4)} - 8y^{(3)} + 29y'' - 40y' + 16y = 0$

(c) $y^{(4)} + 8y^{(3)} + 29y'' + 52y' + 40y = 0$

(d) $y^{(4)} - 8y^{(3)} + 28y'' - 48y' + 32y = 0$

(e) $y^{(4)} - 6y^{(3)} + 20y'' - 40y' + 32y = 0$

15. If the three vectors $v_1 = (1, 2, 3)$, $v_2 = (-1, 0, 4)$, and $v_3 = (3, 2, k)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $\frac{5}{2}$
(b) -5
(c) $-\frac{2}{3}$
(d) 7
(e) -2

16. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' - 2y' = 6e^{2x} + 5\cos x$$

is given by $y_p = Axe^{2x} + B\cos x + C\sin x$. Then $5B + 2A + 5C =$

(a) 0
(b) -3
(c) -9
(d) 5
(e) 7

17. If $c_0 = 1$, $c_k = \dfrac{2k-7}{12k} c_{k-1}, k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of a differential equation about the regular singular point $x = 0$, then the solution is given by

(a) $y = x^{1/2}\left[1 - \frac{5}{12}x - \frac{5}{96}x^2 + \cdots\right]$

(b) $y = x^{1/2}\left[1 - \frac{5}{12}x + \frac{5}{96}x^2 + \cdots\right]$

(c) $y = x^{1/2}\left[1 - \frac{5}{18}x + \frac{5}{96}x^2 + \cdots\right]$

(d) $y = x^{1/2}\left[1 + \frac{5}{12}x + \frac{5}{96}x^2 + \cdots\right]$

(e) $y = x^{1/2}\left[1 - \frac{5}{12}x + \frac{7}{96}x^2 + \cdots\right]$

18. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 2te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & e^{5t} \\ 0 & e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

19. A particular solution of the differential equation $y'' + 4y = 4\sec(2x)$ is given by $y_p(x) =$

(a) $\cos(2x)\ln|\sec(2x) + \tan(2x)| + x\sin(2x)$
(b) $\sin(2x)\ln|\sec(2x) + \tan(2x)| - 1$
(c) $-\cos(2x)\ln|\sin(2x)| + x\sin(2x)$
(d) $2x\cos(2x) + \sin(2x)\ln|\cos(2x)|$
(e) $\cos(2x)\ln|\cos(2x)| + 2x\sin(2x)$

20. The sum of the indicial roots at $x = 0$ for the differential equation

$$x(4-x)y'' + (2-x)y' + 4y = 0$$

is

(a) 0
(b) $\frac{1}{4}$
(c) $-\frac{1}{4}$
(d) $-\frac{1}{2}$
(e) $\frac{1}{2}$

21. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, which has an eigenvalue $\lambda = 2$ of multiplicity 3 and defect 2. If we choose the generalized eigenvector $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ such that $(A - 2I)^3 v_3 = 0$ but $(A - 2I)^2 v_3 \neq 0$, which of the following represents the general solution of the differential system $X' = AX$?

(a) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2/2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(b) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t \\ t^2/2 \\ 1 \end{bmatrix} \right) e^{2t}$

(c) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(d) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t \\ t^2/2 \end{bmatrix} \right) e^{2t}$

(e) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t+1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ 2t \\ 1 \end{bmatrix} \right) e^{2t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 6 **CODE 6**

MATH 208
Final Exam
Term 253
August 5, 2026
Net Time Allowed: 120 Minutes

Name	

ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The sum of the indicial roots at $x = 0$ for the differential equation

$$x(4-x)y'' + (2-x)y' + 4y = 0$$

is

(a) $\frac{1}{2}$
(b) $-\frac{1}{4}$
(c) $\frac{1}{4}$
(d) $-\frac{1}{2}$
(e) 0

2. The solution of the initial-value problem

$$xe^y\,dy + \left(\frac{x^2+1}{y}\right)dx = 0, \qquad y(1) = 1,$$

is given by

(a) $5(y-1)e^y + x^2 - 2\ln|x| = 1$
(b) $3(y-1)e^y + x^2 + 2\ln|x| = 1$
(c) $4(y-1)e^y + x^2 + 3\ln|x| = 1$
(d) $2(y-1)e^y + x^2 + 2\ln|x| = 1$
(e) $(y-1)e^y + x^2 + 2\ln|x| = 1$

3. Consider the system

$$X' = AX, \qquad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

where A is a 2×2 matrix with real entries. If A has the eigenvalue $\lambda = 1 + i$ with corresponding eigenvector $K = \begin{bmatrix} -1 - i \\ 1 \end{bmatrix}$, then $X(\pi) =$

(a) $e^{\pi} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $e^{\pi} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $e^{\pi} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(d) $e^{\pi} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

(e) $e^{\pi} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

4. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' - 2y' = 6e^{2x} + 5\cos x$$

is given by $y_p = Axe^{2x} + B\cos x + C\sin x$. Then $5B + 2A + 5C =$

(a) 5

(b) -9

(c) 7

(d) 0

(e) -3

5. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power-series solution of the differential equation $(x^2+1)y'' - 4xy' + 6y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation

(a) $c_n = \dfrac{(4-n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(b) $c_n = \dfrac{(4-n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(c) $c_n = \dfrac{(4+n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(d) $c_n = \dfrac{(4+n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(e) $c_n = \dfrac{(2+n)(n-5)}{n(n+1)} c_{n-2}, \quad n \geq 2$

6. If the three vectors $v_1 = (1, 2, 3)$, $v_2 = (-1, 0, 4)$, and $v_3 = (3, 2, k)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\dfrac{2}{3}$

(b) -2

(c) -5

(d) $\dfrac{5}{2}$

(e) 7

7. A fundamental matrix of the system $X' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} -e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} 2e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{-t} & 3e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

8. Consider the non-homogeneous system

$$X' = AX + \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If the general solution of the associated homogeneous system is $X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t}$, then a particular solution X_p is

(a) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

9. Suppose that a 2×2 matrix A has the eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$, with corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, respectively. If $A^2 = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, then $x + y + z + w =$

   (Hint: Find a diagonalizing matrix P and write $A = PDP^{-1}$.)

   (a) -6
   (b) -7
   (c) -5
   (d) -10
   (e) -9

10. The minimum radius of convergence of a power-series solution of the differential equation

$$(x^2 - 4x + 13)y'' + 2xy' - y = 0$$

about the ordinary point $x = 1$ is equal to

   (a) 3
   (b) $\sqrt{13}$
   (c) $2\sqrt{13}$
   (d) 5
   (e) $\sqrt{10}$

11. A general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2}{x}y = 4x^3y^4$ is given by

(a) $y^{-3} = 5x^4 + Cx^6$
(b) $y^{-3} = 4x^4 + Cx^6$
(c) $y^{-3} = 2x^4 + Cx^6$
(d) $y^{-3} = 3x^4 + Cx^6$
(e) $y^{-3} = 6x^4 + Cx^6$

12. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & e^{5t} \\ 0 & e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 2te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$

13. The number of irregular singular points of the differential equation

$$x^2(1-x^2)y'' + y' + xy = 0$$

is

(a) 4
(b) 2
(c) 3
(d) 0
(e) 1

14. A linear homogeneous differential equation with real constant coefficients having the solutions xe^{2x} and $3e^{2x}\sin(2x)$ is given by

(a) $y^{(4)} + 8y^{(3)} + 29y'' + 52y' + 40y = 0$
(b) $y^{(4)} - 6y^{(3)} + 20y'' - 40y' + 32y = 0$
(c) $y^{(4)} - 8y^{(3)} + 28y'' - 48y' + 32y = 0$
(d) $y^{(4)} - 8y^{(3)} + 29y'' - 52y' + 40y = 0$
(e) $y^{(4)} - 8y^{(3)} + 29y'' - 40y' + 16y = 0$

15. If the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & -3 & 5 & 4 \\ 3 & -5 & 8 & 5 \\ 1 & -1 & 2 & k^2-6 \end{bmatrix}$ is equal to 2, then $k =$

(a) 0
(b) ± 3
(c) 3 only
(d) $\pm\sqrt{6}$
(e) -3 only

16. The general solution of $X' = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ -2 \\ 1 \end{bmatrix} e^{t}.$$

Then $\alpha + \beta + \gamma =$

(a) 1
(b) -1
(c) -2
(d) 0
(e) 3

17. If $K = \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = -2$ of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & -3 \\ 1 & -4 & 2 \end{bmatrix}$, then $\alpha =$

(a) -2
(b) 1
(c) 2
(d) -1
(e) 0

18. If $c_0 = 1$, $c_k = \dfrac{2k-7}{12k} c_{k-1}, k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of a differential equation about the regular singular point $x = 0$, then the solution is given by

(a) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$
(b) $y = x^{1/2}\left[1 - \dfrac{5}{12}x - \dfrac{5}{96}x^2 + \cdots\right]$
(c) $y = x^{1/2}\left[1 + \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$
(d) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{7}{96}x^2 + \cdots\right]$
(e) $y = x^{1/2}\left[1 - \dfrac{5}{18}x + \dfrac{5}{96}x^2 + \cdots\right]$

19. A particular solution of the differential equation $y'' + 4y = 4\sec(2x)$ is given by $y_p(x) =$

(a) $\cos(2x)\ln|\sec(2x) + \tan(2x)| + x\sin(2x)$
(b) $\sin(2x)\ln|\sec(2x) + \tan(2x)| - 1$
(c) $-\cos(2x)\ln|\sin(2x)| + x\sin(2x)$
(d) $\cos(2x)\ln|\cos(2x)| + 2x\sin(2x)$
(e) $2x\cos(2x) + \sin(2x)\ln|\cos(2x)|$

20. The solution of the exact differential equation

$$\left(\sin y + y\sin x + \frac{1}{x}\right) dx + \left(\frac{1}{y} + x\cos y - \cos x\right) dy = 0$$

is given by

(a) $\ln|x| + \ln|y| + x\cos y - y\cos x = C$
(b) $\ln|x| + \ln|y| + x\sin y - y\cos x = C$
(c) $\ln|x| - \ln|y| - x\cos y - y\cos x = C$
(d) $\ln|x| - \ln|y| + x\sin y - y\cos x = C$
(e) $\ln|x| + \ln|y| + y\sin y - x\cos x = C$

21. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, which has an eigenvalue $\lambda = 2$ of multiplicity 3 and defect 2. If we choose the generalized eigenvector $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ such that $(A - 2I)^3 v_3 = 0$ but $(A - 2I)^2 v_3 \neq 0$, which of the following represents the general solution of the differential system $X' = AX$?

(a) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t+1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ 2t \\ 1 \end{bmatrix} \right) e^{2t}$

(b) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t \\ t^2/2 \end{bmatrix} \right) e^{2t}$

(c) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2/2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(d) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t \\ t^2/2 \\ 1 \end{bmatrix} \right) e^{2t}$

(e) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 7 **CODE 7**

MATH 208
Final Exam
Term 253
August 5, 2026
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
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5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The number of irregular singular points of the differential equation

$$x^2(1-x^2)y'' + y' + xy = 0$$

is

(a) 3
(b) 0
(c) 1
(d) 4
(e) 2

2. Consider the non-homogeneous system

$$X' = AX + \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If the general solution of the associated homogeneous system is $X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t}$, then a particular solution X_p is

(a) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$
(b) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
(c) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$
(d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$
(e) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

3. If the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & -3 & 5 & 4 \\ 3 & -5 & 8 & 5 \\ 1 & -1 & 2 & k^2-6 \end{bmatrix}$ is equal to 2, then $k =$

   (a) 3 only
   (b) 0
   (c) ± 3
   (d) $\pm\sqrt{6}$
   (e) -3 only

4. Consider the system

$$X' = AX, \qquad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

   where A is a 2×2 matrix with real entries. If A has the eigenvalue $\lambda = 1 + i$ with corresponding eigenvector $K = \begin{bmatrix} -1-i \\ 1 \end{bmatrix}$, then $X(\pi) =$

   (a) $e^{\pi} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
   (b) $e^{\pi} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$
   (c) $e^{\pi} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$
   (d) $e^{\pi} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
   (e) $e^{\pi} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

5. If $K = \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = -2$ of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & -3 \\ 1 & -4 & 2 \end{bmatrix}$, then $\alpha =$

   (a) 0
   (b) 2
   (c) -1
   (d) 1
   (e) -2

6. The general solution of $X' = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ -2 \\ 1 \end{bmatrix} e^{t}.$$

   Then $\alpha + \beta + \gamma =$

   (a) 1
   (b) 0
   (c) 3
   (d) -2
   (e) -1

7. A particular solution of the differential equation $y'' + 4y = 4\sec(2x)$ is given by $y_p(x) =$

(a) $-\cos(2x)\ln|\sin(2x)| + x\sin(2x)$
(b) $2x\cos(2x) + \sin(2x)\ln|\cos(2x)|$
(c) $\sin(2x)\ln|\sec(2x) + \tan(2x)| - 1$
(d) $\cos(2x)\ln|\cos(2x)| + 2x\sin(2x)$
(e) $\cos(2x)\ln|\sec(2x) + \tan(2x)| + x\sin(2x)$

8. Suppose that a 2×2 matrix A has the eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$, with corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, respectively. If $A^2 = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, then $x + y + z + w =$
(Hint: Find a diagonalizing matrix P and write $A = PDP^{-1}$.)

(a) -9
(b) -6
(c) -7
(d) -5
(e) -10

9. A fundamental matrix of the system $X' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} -e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{-t} & 3e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} 2e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

10. The sum of the indicial roots at $x = 0$ for the differential equation

$$x(4-x)y'' + (2-x)y' + 4y = 0$$

is

(a) $\frac{1}{2}$

(b) 0

(c) $-\frac{1}{4}$

(d) $\frac{1}{4}$

(e) $-\frac{1}{2}$

11. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' - 2y' = 6e^{2x} + 5\cos x$$

is given by $y_p = Axe^{2x} + B\cos x + C\sin x$. Then $5B + 2A + 5C =$

(a) -9
(b) 5
(c) 0
(d) 7
(e) -3

12. A general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2}{x}y = 4x^3y^4$ is given by

(a) $y^{-3} = 2x^4 + Cx^6$
(b) $y^{-3} = 3x^4 + Cx^6$
(c) $y^{-3} = 5x^4 + Cx^6$
(d) $y^{-3} = 6x^4 + Cx^6$
(e) $y^{-3} = 4x^4 + Cx^6$

13. The solution of the exact differential equation

$$\left(\sin y + y\sin x + \frac{1}{x}\right)\,dx + \left(\frac{1}{y} + x\cos y - \cos x\right)\,dy = 0$$

is given by

(a) $\ln|x| + \ln|y| + y\sin y - x\cos x = C$

(b) $\ln|x| + \ln|y| + x\cos y - y\cos x = C$

(c) $\ln|x| - \ln|y| - x\cos y - y\cos x = C$

(d) $\ln|x| - \ln|y| + x\sin y - y\cos x = C$

(e) $\ln|x| + \ln|y| + x\sin y - y\cos x = C$

14. If $c_0 = 1$, $c_k = \dfrac{2k-7}{12k}c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of a differential equation about the regular singular point $x = 0$, then the solution is given by

(a) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{7}{96}x^2 + \cdots\right]$

(b) $y = x^{1/2}\left[1 - \dfrac{5}{18}x + \dfrac{5}{96}x^2 + \cdots\right]$

(c) $y = x^{1/2}\left[1 + \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$

(d) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$

(e) $y = x^{1/2}\left[1 - \dfrac{5}{12}x - \dfrac{5}{96}x^2 + \cdots\right]$

15. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & e^{5t} \\ 0 & e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & 2te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

16. A linear homogeneous differential equation with real constant coefficients having the solutions xe^{2x} and $3e^{2x}\sin(2x)$ is given by

(a) $y^{(4)} - 8y^{(3)} + 29y'' - 52y' + 40y = 0$

(b) $y^{(4)} - 8y^{(3)} + 28y'' - 48y' + 32y = 0$

(c) $y^{(4)} - 6y^{(3)} + 20y'' - 40y' + 32y = 0$

(d) $y^{(4)} - 8y^{(3)} + 29y'' - 40y' + 16y = 0$

(e) $y^{(4)} + 8y^{(3)} + 29y'' + 52y' + 40y = 0$

17. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power-series solution of the differential equation $(x^2+1)y'' - 4xy' + 6y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation

(a) $c_n = \dfrac{(4+n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(b) $c_n = \dfrac{(4+n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(c) $c_n = \dfrac{(4-n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(d) $c_n = \dfrac{(2+n)(n-5)}{n(n+1)} c_{n-2}, \quad n \geq 2$

(e) $c_n = \dfrac{(4-n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

18. The minimum radius of convergence of a power-series solution of the differential equation

$$(x^2 - 4x + 13)y'' + 2xy' - y = 0$$

about the ordinary point $x = 1$ is equal to

(a) $\sqrt{10}$

(b) $\sqrt{13}$

(c) 3

(d) 5

(e) $2\sqrt{13}$

19. The solution of the initial-value problem

$$xe^y\,dy + \left(\frac{x^2+1}{y}\right)dx = 0, \qquad y(1) = 1,$$

is given by

(a) $5(y-1)e^y + x^2 - 2\ln|x| = 1$
(b) $2(y-1)e^y + x^2 + 2\ln|x| = 1$
(c) $3(y-1)e^y + x^2 + 2\ln|x| = 1$
(d) $(y-1)e^y + x^2 + 2\ln|x| = 1$
(e) $4(y-1)e^y + x^2 + 3\ln|x| = 1$

20. If the three vectors $v_1 = (1,2,3)$, $v_2 = (-1,0,4)$, and $v_3 = (3,2,k)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $-\frac{2}{3}$
(b) $\frac{5}{2}$
(c) -2
(d) -5
(e) 7

21. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, which has an eigenvalue $\lambda = 2$ of multiplicity 3 and defect 2. If we choose the generalized eigenvector $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ such that $(A - 2I)^3 v_3 = 0$ but $(A - 2I)^2 v_3 \neq 0$, which of the following represents the general solution of the differential system $X' = AX$?

(a) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t+1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ 2t \\ 1 \end{bmatrix} \right) e^{2t}$

(b) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t \\ t^2/2 \end{bmatrix} \right) e^{2t}$

(c) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(d) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t \\ t^2/2 \\ 1 \end{bmatrix} \right) e^{2t}$

(e) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2/2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE 8 **CODE 8**

MATH 208
Final Exam
Term 253
August 5, 2026
Net Time Allowed: 120 Minutes

Name			
ID		**Sec**	

Check that this exam has 21 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
3. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
4. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
5. When bubbling, make sure that the bubbled space is fully covered.
6. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The minimum radius of convergence of a power-series solution of the differential equation

$$(x^2 - 4x + 13)y'' + 2xy' - y = 0$$

about the ordinary point $x = 1$ is equal to

(a) $\sqrt{13}$
(b) $\sqrt{10}$
(c) $2\sqrt{13}$
(d) 3
(e) 5

2. If the rank of the matrix $A = \begin{bmatrix} 1 & -2 & 3 & 1 \\ 2 & -3 & 5 & 4 \\ 3 & -5 & 8 & 5 \\ 1 & -1 & 2 & k^2-6 \end{bmatrix}$ is equal to 2, then $k =$

(a) -3 only
(b) 3 only
(c) $\pm\sqrt{6}$
(d) ± 3
(e) 0

3. The number of irregular singular points of the differential equation

$$x^2(1-x^2)y'' + y' + xy = 0$$

is

(a) 1
(b) 3
(c) 4
(d) 2
(e) 0

4. The general solution of $X' = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} X$ can be written as

$$X = c_1 \begin{bmatrix} 1 \\ \alpha \\ 1 \end{bmatrix} e^{4t} + c_2 \begin{bmatrix} 1 \\ 0 \\ \beta \end{bmatrix} e^{-t} + c_3 \begin{bmatrix} \gamma \\ -2 \\ 1 \end{bmatrix} e^{t}.$$

Then $\alpha + \beta + \gamma =$

(a) 0
(b) -2
(c) 3
(d) -1
(e) 1

5. If $A = \begin{bmatrix} 2 & 5 \\ 0 & 2 \end{bmatrix}$, then $e^{At} =$

(a) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & 2e^{2t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{2t} & 2te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{2t} & 5e^{2t} \\ 0 & e^{2t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{2t} & e^{5t} \\ 0 & e^{2t} \end{bmatrix}$

(e) $\begin{bmatrix} e^{2t} & 5te^{2t} \\ 0 & e^{2t} \end{bmatrix}$

6. Suppose that a 2×2 matrix A has the eigenvalues $\lambda_1 = 2$ and $\lambda_2 = 1$, with corresponding eigenvectors $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $v_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, respectively. If $A^2 = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$, then $x + y + z + w =$
(Hint: Find a diagonalizing matrix P and write $A = PDP^{-1}$.)

(a) -10

(b) -6

(c) -9

(d) -5

(e) -7

7. If $y = \sum_{n=0}^{\infty} c_n x^n$ is a power-series solution of the differential equation $(x^2+1)y'' - 4xy' + 6y = 0$ about the ordinary point $x = 0$, then the constants c_n satisfy the recurrence relation

(a) $c_n = \dfrac{(4-n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(b) $c_n = \dfrac{(4-n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(c) $c_n = \dfrac{(4+n)(n-5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

(d) $c_n = \dfrac{(2+n)(n-5)}{n(n+1)} c_{n-2}, \quad n \geq 2$

(e) $c_n = \dfrac{(4+n)(n+5)}{n(n-1)} c_{n-2}, \quad n \geq 2$

8. The solution of the initial-value problem

$$xe^y\,dy + \left(\frac{x^2+1}{y}\right)dx = 0, \qquad y(1) = 1,$$

is given by

(a) $3(y-1)e^y + x^2 + 2\ln|x| = 1$

(b) $5(y-1)e^y + x^2 - 2\ln|x| = 1$

(c) $4(y-1)e^y + x^2 + 3\ln|x| = 1$

(d) $(y-1)e^y + x^2 + 2\ln|x| = 1$

(e) $2(y-1)e^y + x^2 + 2\ln|x| = 1$

9. A general solution of the differential equation $\dfrac{dy}{dx} + \dfrac{2}{x}y = 4x^3y^4$ is given by

(a) $y^{-3} = 3x^4 + Cx^6$
(b) $y^{-3} = 5x^4 + Cx^6$
(c) $y^{-3} = 4x^4 + Cx^6$
(d) $y^{-3} = 6x^4 + Cx^6$
(e) $y^{-3} = 2x^4 + Cx^6$

10. If $K = \begin{bmatrix} \alpha \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = -2$ of the matrix $A = \begin{bmatrix} 0 & 1 & -1 \\ -3 & 1 & -3 \\ 1 & -4 & 2 \end{bmatrix}$, then $\alpha =$

(a) 0
(b) -2
(c) 2
(d) 1
(e) -1

11. If $c_0 = 1$, $c_k = \dfrac{2k-7}{12k} c_{k-1}$, $k \geq 1$, is the recurrence relation corresponding to the indicial root $r = \dfrac{1}{2}$ in the series solution of a differential equation about the regular singular point $x = 0$, then the solution is given by

(a) $y = x^{1/2}\left[1 + \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$

(b) $y = x^{1/2}\left[1 - \dfrac{5}{12}x - \dfrac{5}{96}x^2 + \cdots\right]$

(c) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{7}{96}x^2 + \cdots\right]$

(d) $y = x^{1/2}\left[1 - \dfrac{5}{12}x + \dfrac{5}{96}x^2 + \cdots\right]$

(e) $y = x^{1/2}\left[1 - \dfrac{5}{18}x + \dfrac{5}{96}x^2 + \cdots\right]$

12. A particular solution of the differential equation $y'' + 4y = 4\sec(2x)$ is given by $y_p(x) =$

(a) $2x\cos(2x) + \sin(2x)\ln|\cos(2x)|$

(b) $\cos(2x)\ln|\cos(2x)| + 2x\sin(2x)$

(c) $\cos(2x)\ln|\sec(2x) + \tan(2x)| + x\sin(2x)$

(d) $-\cos(2x)\ln|\sin(2x)| + x\sin(2x)$

(e) $\sin(2x)\ln|\sec(2x) + \tan(2x)| - 1$

13. By using the method of undetermined coefficients, a particular solution of the differential equation

$$y'' - 2y' = 6e^{2x} + 5\cos x$$

is given by $y_p = Axe^{2x} + B\cos x + C\sin x$. Then $5B + 2A + 5C =$

(a) -9
(b) 0
(c) 7
(d) -3
(e) 5

14. The sum of the indicial roots at $x = 0$ for the differential equation

$$x(4-x)y'' + (2-x)y' + 4y = 0$$

is

(a) $-\frac{1}{4}$
(b) 0
(c) $\frac{1}{2}$
(d) $-\frac{1}{2}$
(e) $\frac{1}{4}$

15. Consider the system

$$X' = AX, \qquad X(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$

where A is a 2×2 matrix with real entries. If A has the eigenvalue $\lambda = 1 + i$ with corresponding eigenvector $K = \begin{bmatrix} -1 - i \\ 1 \end{bmatrix}$, then $X(\pi) =$

(a) $e^{\pi} \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

(b) $e^{\pi} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(c) $e^{\pi} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(d) $e^{\pi} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(e) $e^{\pi} \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

16. The solution of the exact differential equation

$$\left(\sin y + y \sin x + \frac{1}{x}\right) dx + \left(\frac{1}{y} + x \cos y - \cos x\right) dy = 0$$

is given by

(a) $\ln|x| + \ln|y| + x \cos y - y \cos x = C$

(b) $\ln|x| - \ln|y| - x \cos y - y \cos x = C$

(c) $\ln|x| + \ln|y| + x \sin y - y \cos x = C$

(d) $\ln|x| - \ln|y| + x \sin y - y \cos x = C$

(e) $\ln|x| + \ln|y| + y \sin y - x \cos x = C$

17. Consider the non-homogeneous system

$$X' = AX + \begin{bmatrix} 0 \\ 3 \end{bmatrix}.$$

If the general solution of the associated homogeneous system is $X_c = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t}$, then a particular solution X_p is

(a) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

18. A fundamental matrix of the system $X' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} X$ is

(a) $\begin{bmatrix} -e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(b) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{3t} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-t} & e^{3t} \\ e^{-t} & e^{3t} \end{bmatrix}$

(d) $\begin{bmatrix} e^{-t} & 3e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

(e) $\begin{bmatrix} 2e^{-t} & e^{3t} \\ -e^{-t} & e^{3t} \end{bmatrix}$

19. If the three vectors $v_1 = (1,2,3)$, $v_2 = (-1,0,4)$, and $v_3 = (3,2,k)$ of $\mathbb{R}^3$ are linearly dependent, then $k =$

(a) $\frac{5}{2}$
(b) $-\frac{2}{3}$
(c) 7
(d) -2
(e) -5

20. A linear homogeneous differential equation with real constant coefficients having the solutions xe^{2x} and $3e^{2x}\sin(2x)$ is given by

(a) $y^{(4)} + 8y^{(3)} + 29y'' + 52y' + 40y = 0$
(b) $y^{(4)} - 8y^{(3)} + 29y'' - 40y' + 16y = 0$
(c) $y^{(4)} - 6y^{(3)} + 20y'' - 40y' + 32y = 0$
(d) $y^{(4)} - 8y^{(3)} + 28y'' - 48y' + 32y = 0$
(e) $y^{(4)} - 8y^{(3)} + 29y'' - 52y' + 40y = 0$

21. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, which has an eigenvalue $\lambda = 2$ of multiplicity 3 and defect 2. If we choose the generalized eigenvector $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ such that $(A - 2I)^3 v_3 = 0$ but $(A - 2I)^2 v_3 \neq 0$, which of the following represents the general solution of the differential system $X' = AX$?

(a) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ t \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t \\ t^2/2 \\ 1 \end{bmatrix} \right) e^{2t}$

(b) $X(t) = \left(c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ t \\ t^2/2 \end{bmatrix} \right) e^{2t}$

(c) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2/2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(d) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ t \\ 1 \end{bmatrix} \right) e^{2t}$

(e) $X(t) = \left(c_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} t+1 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} t^2 \\ 2t \\ 1 \end{bmatrix} \right) e^{2t}$

MATH 208, Term 253, Final Exam – Answer Key

Q	MASTER	5	6	7	8
1	A	D 13	A 4	C 7	B 16
2	A	B 9	D 3	E 13	D 5
3	A	E 1	E 9	C 5	A 7
4	A	C 19	B 14	B 9	E 19
5	A	B 5	B 8	A 10	E 18
6	A	B 2	C 12	A 19	E 6
7	A	C 8	B 11	D 15	A 8
8	A	B 3	D 13	C 6	E 3
9	A	A 6	B 6	A 11	D 2
10	A	A 16	E 16	A 4	A 10
11	A	A 10	E 2	A 14	D 17
12	A	D 7	D 18	D 2	B 15
13	A	A 11	E 7	E 1	A 14
14	A	D 20	C 20	D 17	C 4
15	A	B 12	B 5	C 18	A 9
16	A	C 14	A 19	B 20	C 1
17	A	B 17	E 10	C 8	B 13
18	A	A 18	A 17	A 16	A 11
19	A	E 15	D 15	B 3	E 12
20	A	E 4	B 1	D 12	D 20
21	A	A 21	C 21	E 21	C 21

Answer Counts

V	A	B	C	D	E
5	6	6	3	3	3
6	3	6	3	4	5
7	6	3	5	4	3
8	6	3	3	4	5

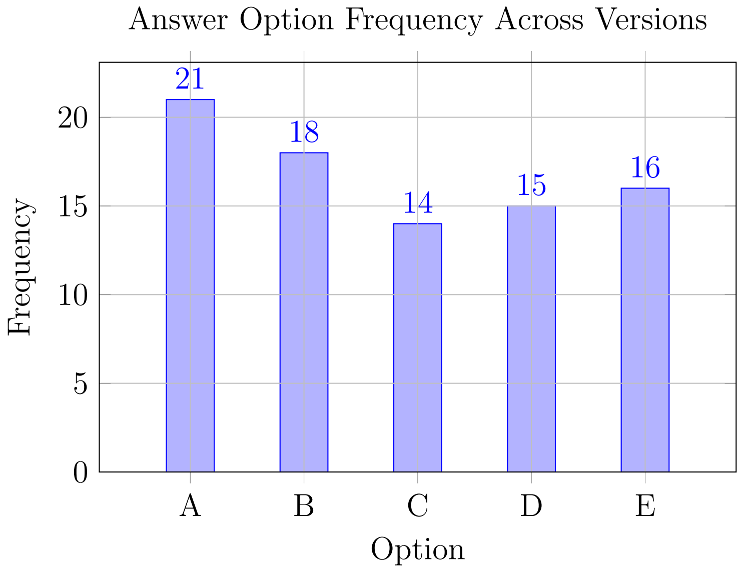

Figure 1: Frequency of each answer option, excluding the master.